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Qualification of the Profilometer

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1. Introduction

The profilometer was purchased from Anorad Corporation in 1988 (Anorad, Hauppauge, NY 11788, 516-231-1990). Data acquisition software (Tucker, 1993) reads the X and Z counts of a profile. Data analysis software (Allen, 1992) converts counts to inches, corrects for a finite probe-tip radius, and removes the profilometer beam shape. The results are then fit to a conic using the program Curvmon written by Harland Epps.

In January 1994 Anorad completed the first of a two-phase upgrade. This phase converted the analog control of the carriage to digital control and resulted in an improved carriage positioning (Mast, 1994). This month Anorad completed the final phase. They replaced the steel probe shaft with a Zerodur probe shaft and provided the option for future use of a built-in reference flat.

Section 2 describes the mechanical properties of the profilometer. Sections 3 and 4 discuss the sensitivities to changing loads and to changing temperatures.

The profilometer is a comparison device; comparing the profile of an unknown optic with the profile of a reference surface known from interferometry. This comparison is made in two steps. First, the reference surface is used to establish the profile of the "beam", i.e. the carriage air-bearing surface on the bridge. Sometimes our reference optics are not large enough to span the entire beam and we stitch together measurements of the reference surface made at different positions along the beam. Then the beam profile is subtracted from the measured profile of the unknown optic. The measurement process can be described by the following relations between profiles.

P(optic) = P(meas) – P(beam)

P(beam) = Stitch[P(ref_meas) – P(ref_interfer)]

Sections 5 and 6 describe the statistical and systematic errors in the measured profiles; P(meas) and P(ref_meas). The errors in the known reference profile, P(ref_interfer), are described in Section 7 and Appendix 10. The errors in the stitched P(beam) are described in Section 9 and Appendix 10. The errors are summarized in Section 9. Profiles of different diameters are combined to give a surface measurement, and the errors in this surface measurement are also described in Section 10.

Our optics shop traditionally uses English units. For communication with others, we typically need metric units. We have tried to include both units in this report, particularly for surface height values (micro-inches and microns).

In practice the smallest aberrations of interest will depend on the stage of fabrication. However, even in the final polishing we are not trying to achieve perfection. We define a smallest level of interest:

$\approx$ 0.2 micro-inches $\approx$ 0.005 microns.

The carriage position (X) and the surface height (Z) are measured in digital counts. The approximate conversion is one count in X = 0.42 micro-inches = 0.0106 microns and one count in Z = 0.21 micro-inches = 0.0053 microns. An accurate conversion is used in the data analysis.

The profilometer purchase and the upgrades were funded by C.A.R.A. for the fabrication of the secondary mirrors for the Keck telescopes. The qualification described here is part of the fabrication of the Keck 2 secondary, and we use the f/15 mirror as the main example of the sizes and the effects of errors.

Warning: The effects of the errors and sensitivities on the final surface measurement will be different for each optic, and they need to be re-considered separately for each optic.

2. Mechanical Properties of the Profilometer

A schematic layout is shown in Figure 1. The profilometer and the optic being measured rest on a granite table supported by a steel frame. The steel frame is isolated from the ground by Barry mounts under the four legs of the steel frame. Two granite blocks on the table support a granite bridge. A carriage is driven along the bridge (X-axis) by a linear motor and carries a vertically moving Zerodur shaft with a ruby ball contact probe. The carriage position (X) and the surface height (Z) are measured using separate Hewlett-Packard fringe-counting laser interferometers. Table 1 of Appendix 1 lists some properties of materials. Table 2 of Appendix 1 lists mechanical parameters of the profilometer components. Table 3 of Appendix 1 lists the size and weight of some of the optics used in this qualification program.

Air bearings are used to float the carriage on the bridge and to laterally support the probe shaft in the assembly. Air pressure is also used to lift (retract) the probe shaft when it is not resting on the optic. Figure 2 shows the air pressure system used to control these tasks.

Figure 3 shows the optical layout of the interferometer systems.

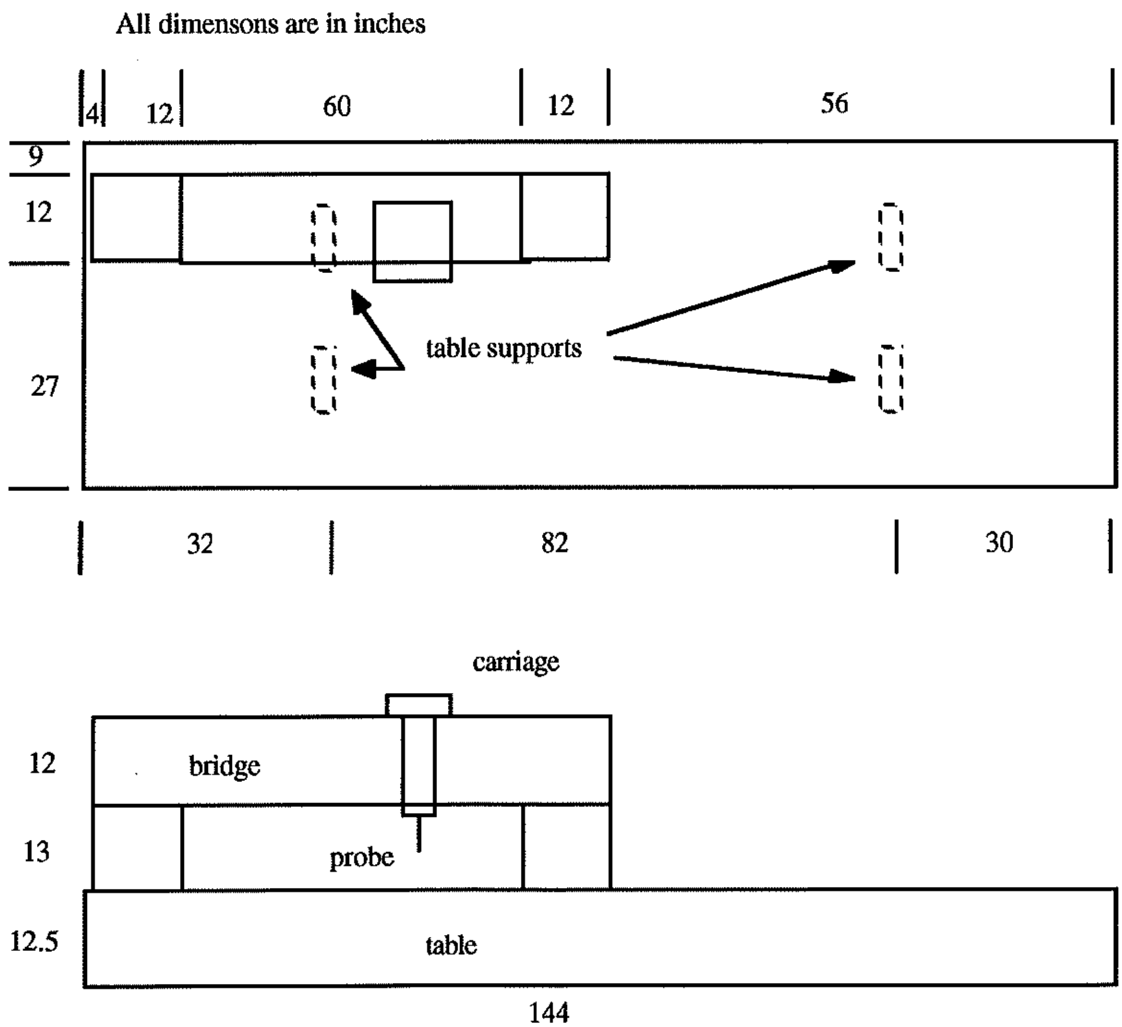


Figure 1. Profilometer Layout

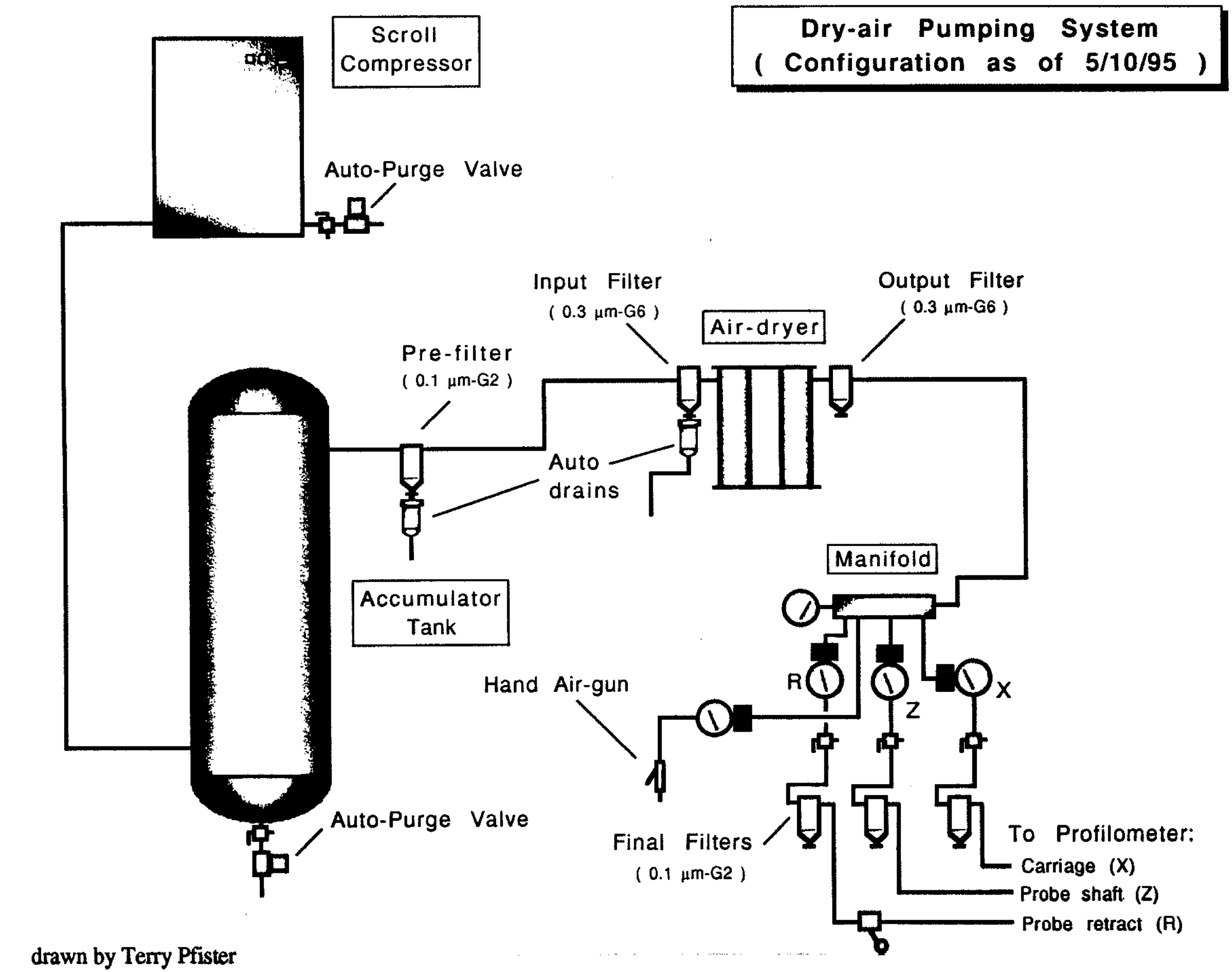


Figure 2. Pressure System for Air Bearings and Probe Retraction

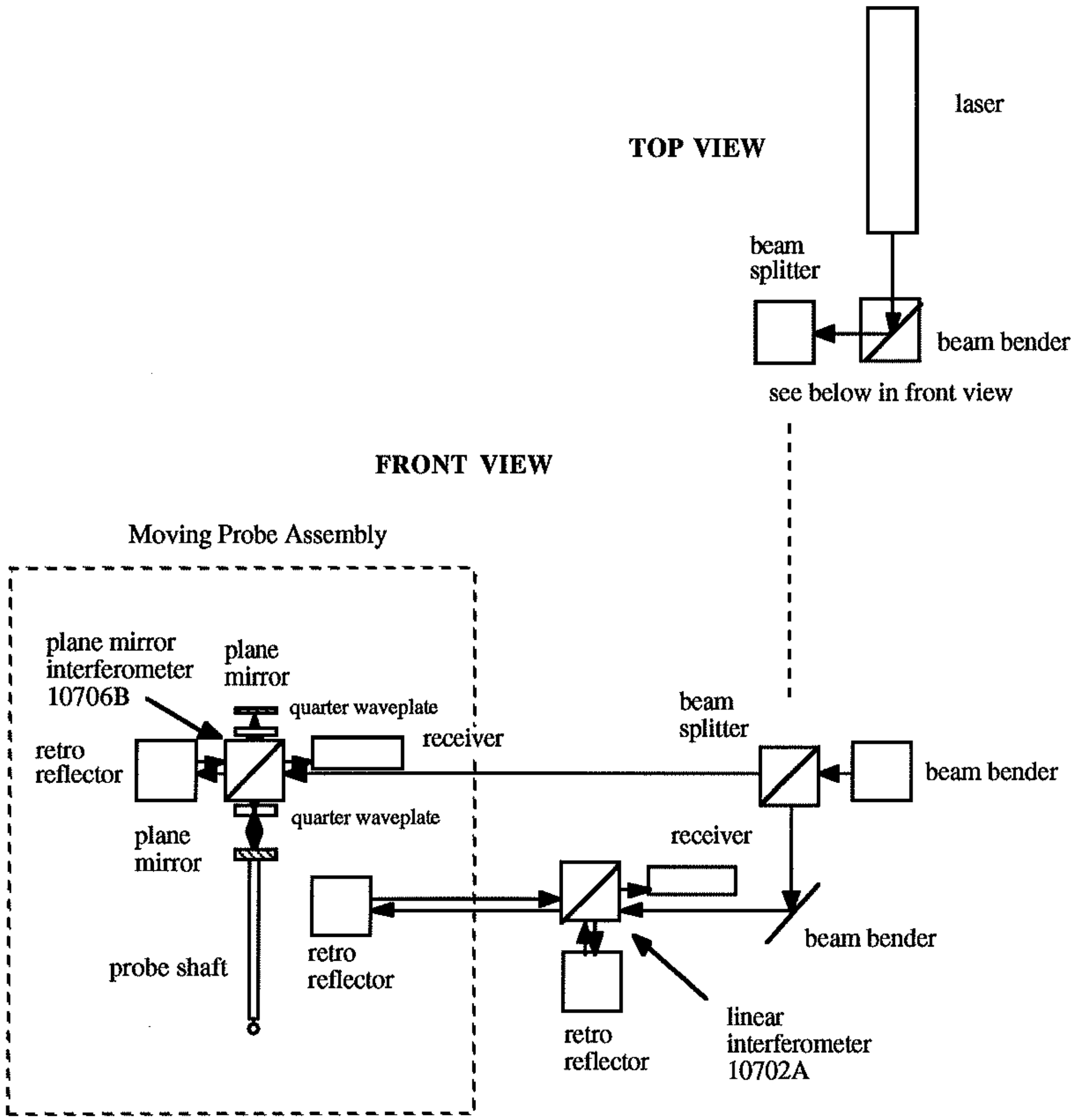


Figure 3. Profilometer Optical Layout

3. Sensitivities to Loads

We calculate here the effect of static loads in order to know the size of some effects that we do not expect to change. We also calculate the effects of changing loads, for example due to interchanging optics and carriage motion. The spirit of this section is to estimate the magnitude of the effects. We use simple formulas that roughly describe the physical structure. The detailed equations and calculations are given in Appendix 2. The results are summarized in Table 1 below.

Table 1. Sensitivities to Loads

	maximum deflection (micro-inches/microns)	slope at supports (micro-radians)	resonant frequency (Hz)
Self-Weight Load of the bridge	−270 / −6.8	12	
Carriage Load on the Bridge	−45 / −1.1	1.9	470
Carriage Load on the Air Bearings			310
Loads on the Granite Table (Changing 40-inch flat position)	−81 / −2.05	−3.5	
Profilometer on Barry Mounts			2.2

Vertical Loads on the Probe Shaft

Assumed force of probe on glass P = 1.0 ± 0.1 grams
Contact Area Radius = 180 micro-inches = 4.5 microns
Maximum Contact Pressure (at center of contact) = 3.2 x 10^4 psi
Total Deformation of both surfaces = 0.8 micro-inches = 0.028 microns

Most of the above effects are static and do not affect the measurements. Only the changing position of the 40-inch flat during the beam-shape measurement has a potential for introducing errors. At the extreme this induces a change in the tilt on the bridge support blocks of 3.5 micro-radians. This induces a sag in the bridge, resulting in quadratic errors in the profiles used for the stitching.

The self-weight sag of the bridge is 6.8 microns over a span of L = 72 inches. As an extreme we consider the case where the rotation of the support blocks shifts the support to their outer edges, L =84 inches. This increases the bridge sag by 59% or 4.0 microns. Over the measured span of the 40-inch flat (38 inches) this is a sag of 1.1 microns. However, the tilt of the blocks (3.5 micro-radians) is small compared to the self-weight slopes of the bridge (12 micro-radians). Thus we expect the actually effect to be smaller than this extreme. If the effective support point only moves ~1/4 of the distance, so Leff = 75 inches, then the effect is reduced by a factor of ~3 to a sag change of about 0.3 microns. Even this may still dominate the systematic errors in the beam shape. However, without a detailed finite element analysis, the actual size of this effect is unknown. Since our estimates have been based on extreme cases and don't include the averaging of the multiple profiles, we assume a more detailed calculation would show this effect to be negligible.

4. Sensitivities to Temperature Changes

Three temperatures (^oF) are currently measured with each profile and recorded in the data file:

TEMP0 - ambient air above the right end of the bridge
TEMP1 - water bath that thermalizes the compressed air
TEMP2 - linear motor steel rail on the bridge (measured at the right end)

Temperature variations induce changes in

1. the tilt and piston of the optic on the table
2. the tilt and piston of the bridge
3. the curvature of the bridge
4. steel and air in the components defining the X-axis counts
5. steel and air in the components defining the Z-axis counts

We estimate the size of each of these effects. The equations and calculations are given in Appendix 3. As will be seen below the systematic errors in a profile measurement are dominated by temperature-induced changes in the bridge curvature. Measurements of this effect have been made many times, and in Appendix 4 we describe some of the results. Table 2 below summarizes the sensitivities.

Table 2. Sensitivities to Temperature Changes

Piston and tilt of the optic on the table

Thermal expansion of the Keck secondary aluminum base

Piston = 69 micro-inches = 1.8 microns / ^oF

Piston and tilt of the bridge

Thermal expansion of both support blocks

Bridge Piston = 58 micro-inches = 1.5 microns / ^oF

Thermal expansions of one block

Bridge Tilt = 0.8 micro-radians / ^oF

Curvature of the bridge

The linear motor is a stack of a steel beam and aluminum U-channel. These are coupled along their length to the granite beam. Temperature changes induce a "bi-metal" (actually tri-metal) curvature change in the granite beam.

The sag has been measured many times in the process of measuring surfaces. Appendix 4 describes those measurements of the f/15 mirror with R = 28 inches. The measurements yield.

f/15 measured change in radius of curvature

δk = 5100 micro-inches / ^oF = −130 microns / ^oF

This corresponds to a change in sag (over R = 28 inches)

δSAG = 60 micro-inches / ^oF = 1.51 microns / ^oF.

We have also calculated the expected sag from the "bimetallic" construction of the bridge and linear motor, using Timeshenko (1925) (see Appendix 4). We calculate a sag of 37 micro-inches / ^{o}F over a span of 60 inches. This is about a factor of two lower than the measured sensitivity, and we do not understand the origin of this discrepancy.

X-axis

Steel:

Thermal expansion during a measurement generates a cubic in the profile

$$\delta a_3 = 2.4 \times 10^{-7} \ / \ ^{o}F \ \frac{a^2}{2k}$$

For the Keck secondary this is $\delta a_3 = 0.0132$ microns / ^{o}F.

Air:

Thermal expansion during a measurement generates a cubic in the profile

$$\delta a_3 = 8.7 \times 10^{-7} \ / \ ^{o}F \ \frac{a^2}{2k}$$

For the Keck secondary this is $\delta a_3 = 0.048$ microns / ^{o}F.

Z-axis

Steel/Zerodur:

Thermal expansion during a measurement generates constant and linear terms in the profile (+2.1 micro-inches / ^{o}F = 0.053 microns / ^{o}F) and these are removed in the data analysis.

Air:

Thermal expansion during a measurement generates constant and linear terms in the profile (+1.4 micro-inches / ^{o}F = 0.037 microns / ^{o}F) and these are removed in the data analysis.

5. Statistical Errors in a Profile

The profilometer can be operated in one of three modes:

1) Lift and Set, where the probe is retracted between moves in X.
2) Stop and Read, where the probe remains on the surface and the X-motion is stopped before the Z position in sampled.
3) Read While Moving, the X-motion is continuous, and the Z position is sampled rapidly at pre-determined positions.

We have traditionally used the Stop and Read mode, and all measurements made for this qualification were taken in that mode. The errors achieved by the other modes may be larger or smaller and remain to be determined by a future program.

The statistical errors were measured using repeated measurements of three reference surfaces; the 40-inch flat, the 20-inch flat, and the 22-inch sphere. Descriptions of interferometric measurements of these surfaces are given in Appendix 5. However, the surface shapes are not important for the repeatability measurements described here. The results are summarized in Table 3.

Repeated measurements of the same profile were also made of the f/15 secondary, although the mirror was rotated between measurements.

40-inch flat

On 15 September 94 we measured the beam shape three times (files keck/2/se15f, g, and h). The 40-inch flat was positioned at three different locations along the beam. Averaging the three repeat measurements and comparing the deviations of each profile with the average yields a measure of the statistical noise.

X-values

The rms deviation of the three measurements from the mean is

1.1 counts = 11 nanometers = 0.45 micro-inches.

Z-values

Constant, linear, and quadratic terms were removed from the profiles. We ascribe the quadratic variations to the bridge curvature (see Section 4). The rms deviation from average of the three repeat profiles is

2.2 counts = 12 nanometers = 0.46 micro-inches.

There is no apparent structure in the deviation profiles.

20-inch flat

On 16 September 94 we measured the 20-inch flat multiple times (3 to 4) at five different positions along the beam. Averaging the multiple measurements and comparing the deviations of each profile with the average yields measures of the statistical noise.

X-values

The rms deviation of the three measurements from the mean is

1.1 counts = 12 nanometers = 0.45 micro-inches.

Z-values

Constant and linear terms were removed from the profiles. The rms deviation of the multiple repeats from the average is

1.59 counts = 8.4 nanometers = 0.33 micro-inches.

There is no apparent structure in the deviation profiles.

22-inch sphere

On 23 September 94 we measured a single profile of the 22-inch sphere four times. Averaging the multiple measurements and comparing the deviations of each profile with the average yields measures of the statistical noise.

X-values

The rms deviation of the three measurements from the mean is

1.03 counts = 11 nanometers = 0.43 micro-inches.

Z-values

Constant, linear, and quadratic terms were removed from the profiles. The rms deviation of the multiple repeats from the average is

3.84 counts = 20 nanometers = 0.80 micro-inches.

There is some structure in the deviation profiles, although not with any obvious repeated functional dependence.

f/15 secondary

On 21 September 94 we measured a profile of the f/15 mirror five times. The scans were made at 0, 180, 0, 180, and 0 degrees successively, rotating the mirror 180 degrees between measurements. Averaging the multiple measurements and comparing the deviations of each profile with the average yields measures of the statistical noise.

X-values

The rms deviation of the three measurements from the mean is

1.24 counts = 13 nanometers = 0.51 micro-inches.

Z-values

Constant, linear, and quadratic terms were removed from the profiles. The rms deviation of the multiple repeats from the average is

3.21 counts = 17 nanometers = 0.67 micro-inches.

There is some structure in the deviation profiles, although not with any obvious repeated functional dependence. We ascribe these variations to errors in the angular positioning between the profiles combined with the azimuthal variations in the mirror. These dominate the rms deviations, and so we do not include these values in the table below.

Table 3. Summary of Measurements of Statistical Errors.

	X (μ-inches)	X (microns)	Z (μ-inches)	Z (microns)
40-inch flat	0.45	0.011	0.46	0.012
20-inch flat	0.45	0.012	0.33	0.008
22-inch sphere	0.43	0.011	0.80	0.020
Total rms	0.44	0.011	0.56	0.014

The effect of statistical errors on a fit to the profile.

If one fits a series of points (x_i, z_i) to

$$z_i = a_2 \left(\frac{x_i}{a}\right)^2 + a_4 \left(\frac{x_i}{a}\right)^4$$

and propagates Gaussian errors (σ) in z_i into errors in the coefficients gives

$$\delta a_2 = \frac{4\sqrt{5}}{\sqrt{N}}\,\sigma \qquad \delta a_4 = \frac{48}{\sqrt{N}}\,\sigma .$$

For a conic,

$$a_{20} = \frac{a^2}{2k} \qquad \text{and} \qquad a_{40} = \frac{(K+1)\,a^4}{8k^3}$$

Using the inverse of these to approximate k and K gives statistical errors

$$\delta k = \frac{2k^2}{a^2}\,\frac{\sigma 4\sqrt{5}}{\sqrt{N}} \qquad \delta K = \frac{8k^3}{a^4}\,\frac{\sigma 48}{\sqrt{N}}$$

Example: Keck Secondary ($a = 0.723$ meters, $k = -4.738$ meters) and $N = 112$

If we consider only one profile and assume the errors in P(beam) are zero, then $\sigma = 0.014$ microns, and

δa_2 = 0.46 micro-inches = 0.012 microns
δa_4 = 2.5 micro-inches = 0.064 microns .
δk = 34 micro-inches = 1.03 microns
δK = 0.00020

For the Keck secondary these are both negligible compared to the tolerances; $\delta k = \pm 5000$ microns and $\delta K = 0.0008$.

6. Systematic Errors in a Profile

We consider the following sources of systematic errors.

Alignment Errors
Temperature-Induced Errors
Gravity-Induced Errors
Counts-to-Inches Conversion Errors
Probe-Tip Correction Errors

Alignment Errors

If the x-axis and z-axis of the profilometer are not perpendicular, then a systematic error in introduced into each profile. To lowest order this is a cubic term (Appendix 7). If the angle between the two axes differs by θ from $\pi/2$, then the cubic term generated in the profile is

$$z = a_3 \frac{x^3}{a^3} \qquad \text{where} \quad a_3 = \frac{a^3}{2k^2}\,\theta ,$$

k = the radius of curvature and a = the radius or half-diameter of the optic.

For example: On 20 Sept 94 we measured a test sphere (k = 0.347 meters and a = 0.146 meters) and obtained

$$\theta = a_3 \frac{2k^2}{a^3} = 18 \text{ arc seconds.}$$

Temperature-Induced Errors

Piston and tilt are removed from each profile by Curvmon. The next order term, the quadratic term in each profile dominates the determination of the radius of curvature of the measured optics.

The systematic error in the quadratic term is dominated by the temperature-induced change in the bridge curvature. In Section 4 and Appendix 4 we show this gives an absolute error in each profile of

$$\delta k = -5100 \text{ micro-inches} / {}^{o}F = -130 \text{ microns} / {}^{o}F$$

Although we are currently measuring the rail temperature, we are not correcting each profile to a standard temperature.

Gravity-Induced Errors

As the carriage moves across the bridge, its weight causes a deflection in the bridge. In Section 3 above we estimate the maximum sag is 45 micro-inches (1.1 microns) This causes an error in the sag of the measured profiles, P(meas) and P(meas_ref). Since we subtract these two to calculate P(optics) this effect cancels.

The most important effect is that of moving the 40-inch reference flat during the measurement of the beam shape. This deforms the granite table, tilting the bridge support blocks, and bending the bridge. We have not made a detailed calculation of this effect. A rough estimate suggests it is less than about 0.1 microns of rms error in the stitched profile.

Counts-to-Inches Conversion Errors

Errors in the conversion from counts to inches (Appendix 5) are negligible.

Probe-Tip Correction Errors

Errors in the probe-tip correction (Appendix 6) are negligible.

7. Errors in the Reference Surface

We currently use one or more of three reference surfaces to establish P(ref_interfer). Detailed discussions of the measurements of these surfaces are given in Appendix 10. We summarize those results in the table below.

Table 4. Summary of Some Reference Surface Parameters

	span (inches)	sag ± error in sag (micro-inches)	surface rms (micro-inches)	surface rms (microns)
20-inch flat	20	2.5 ± 0.3	0.1	0.003
40-inch flat	40	42.5 ± 2.5	< 2.4	< 0.060

	span (inches)	sag ± error in sag (inches)	surface rms (micro-inches)	surface rms (microns)
22-inch sphere	20	0.33800 ± 0.00001	< 0.6	< 0.005

8. Carriage-Height and -Tilt Correction Errors

The air-bearing surface of the granite bridge is not perfectly flat As the carriage moves in X, both it's height and tilt change by small amounts.

The variations of the height, $Z_{carriage}(X)$, we call the "carriage-height profile"

The variations of the tilt, $\Theta_{carriage}(X)$, we call the "carriage-tilt profile."

The quantity P(ref_meas) (in Section 1 above) is calculated from both $Z_{carriage}(X)$ and $\Theta_{carriage}(X)$. We can write

$$P(ref\_meas) = P_Z(ref\_meas) + P_\Theta(ref\_meas)$$

Note: In the past "beam.shape" referred to the carriage-height profile.

Carriage-Height Correction

The carriage-height profile is measured using a known reference flat. This has been a standard part of the measurement and analysis procedures and results in the file called "beam.shape" which is subtracted from the measured heights of the part.

$$P_Z(ref\_meas) = Z_{carriage}(X)$$

We report here an example using the 20-inch flat measured at five positions along the bridge. We measured the carriage-height profile nine times with 1/8-inch spacing at the same sample points used to measured the Keck 2 secondary.

	file	ΔT (^oF)	rms res (micro-inches)	mean T (^oF)
1	fe21f	0.23	0.14	70.79
2	fe21g	0.56	0.59	70.98
3	fe21h	0.62	0.61	71.02
4	ma15g	0.43	0.58	69.53
5	ma15h	0.41	0.49	69.56
6	ma15i	0.38	0.44	69.58

7	ma20f	0.23	0.31	70.50
8	ma20g	0.11	0.19	70.62
9	ma20h	0.23	0.17	70.64

During each of the nine measurements the rail temperature varied over a range ΔT. Plotting the rms residual from the splicing versus the range of rail temperature shows a good correlation. Since the splicing does not account for changes in curvature, the rms residual increases with larger variations in the curvature from position to position.

We averaged the resulting bridge profiles, calculated a mean profile, and calculated the deviation profiles; the difference between the individual profiles and the mean. These differ by quadratics, as expected, since the mean temperature of each measurement was different. We removed the quadratic component from each of the profiles. Three of the quadratic-removed, deviation profiles showed systematic deviations and had larger rms deviations (fe21g,ma15i, and ma20f). We eliminated these three and averaged the remaining six. The rms deviation of the six quadratic-removed, deviation profiles is 1.27 micro-inches. For the final carriage-height correction we use the mean of the six carriage-height profiles (fe21f,fe21h,ma15g,ma15h,ma20g,ma20h) and estimate the error on each point to be $1.27 / \sqrt{6} = 0.5$ micro-inches.

Figure 4 shows the carriage-height correction profile. It varies over a range of about 100 micro-inches, and there is a distinct difference between the shape on the negative and positive X portions. There is a short periodicity of about 2.2 inches. On the positive X portion there is a larger wavelength periodicity superimposed. The origins of these are not understood.

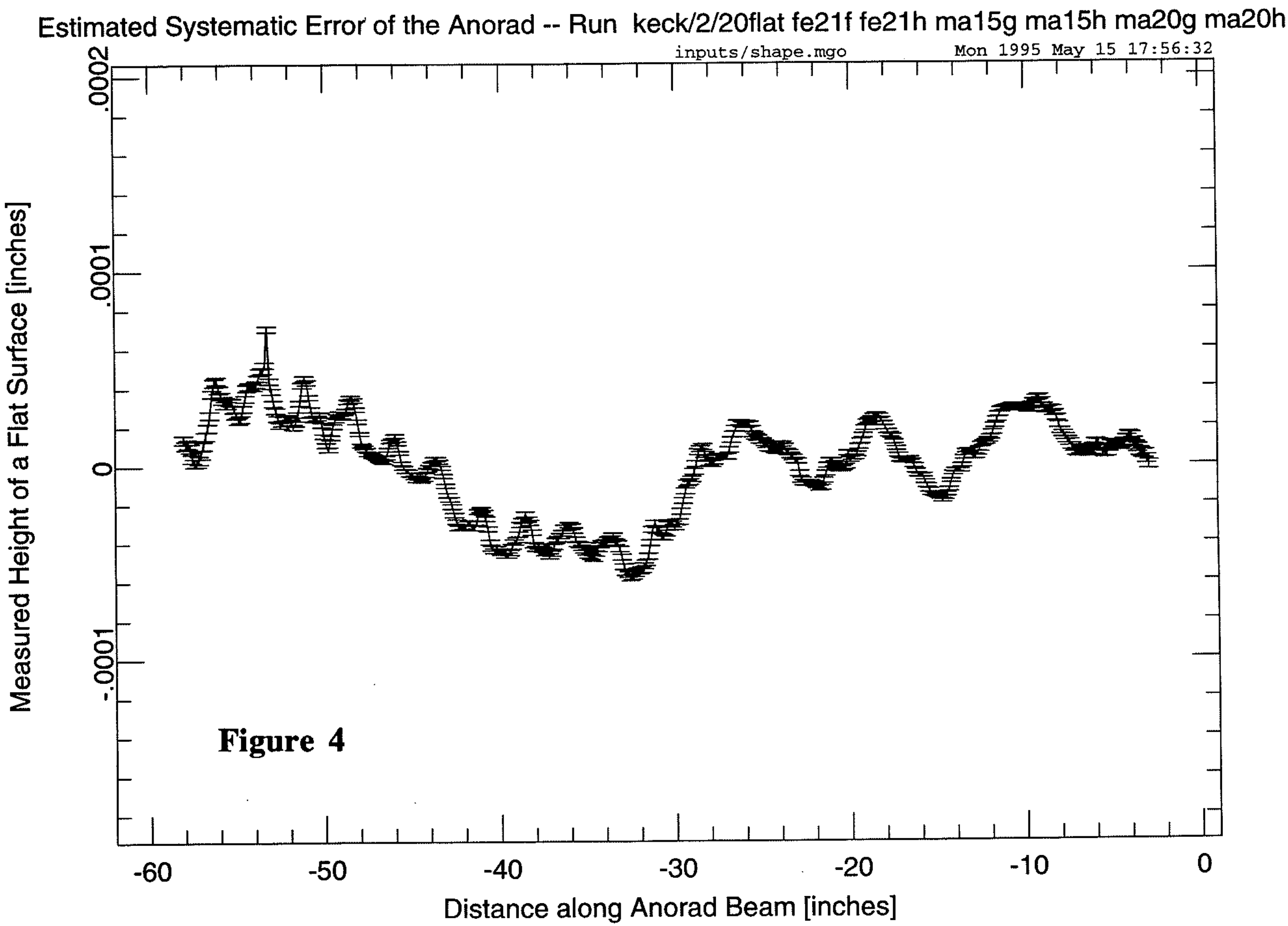

Estimated Systematic Error of the Anorad -- Run keck/2/20flat fe21f fe21h ma15g ma15h ma20g ma20h
inputs/shape.mgo
Mon 1995 May 15 17:56:32
.0002
.0001
0
-.0001
Measured Height of a Flat Surface [inches]
-60
-50
-40
-30
-20
-10
0
Distance along Anorad Beam [inches]

Figure 4

If we want measurements accurate to 1 micro-inch, this correction needs to be determined and removed with an accuracy of better than 1%. In addition, the quadratic variations in this correction depend on a model of the temperature variations. This correction (with the high required accuracy) will be unnecessary when we implement the overhead flat option to the profilometer.

Carriage-Tilt Correction

The carriage-tilt profile also affects the measured surface height as follows. A tilt of the carriage displaces the X-position of the probe tip. If the surface being measured is not horizontal, then the X displacement will cause the ball to drop or be pushed up, creating a change in the measured height. The size of the change depends on 1) the carriage tilt, 2) the lever arm between the probe-assembly rotation point and the probe tip, and 3) the slope of the surface being measured.

The carriage has two pads, each of length P and separated by a distance C.

One might think we could calculate the carriage-tilt profile from the carriage-height profile. This cannot be done since some Fourier components of the granite bearing profile have no effect on the height profile, but can have a large effect on the tilt profile. For example, consider a bearing profile that is a sine wave with wavelength equal to C/2. The average of the two pad heights (the carriage height) is constant. The difference of the pad heights (carriage tilt * C) varies by twice the amplitude of the sine wave. We conclude the tilt-profile must be independently measured.

We have used a separate Hewlett-Packard distance-measuring system to measure the carriage tilt. We set a retroreflector at the top of the probe assembly about 14 inches above the retroreflector used to control the carriage X-position. This X-control height is the effective carriage assembly rotation point. The difference in the two measurements divided by the 14 inches, gives the carriage tilt. The resulting carriage-tilt profile is shown in Figure 5. The tilt ranges over about 9 arc seconds.

carr-tilt ma15 ch

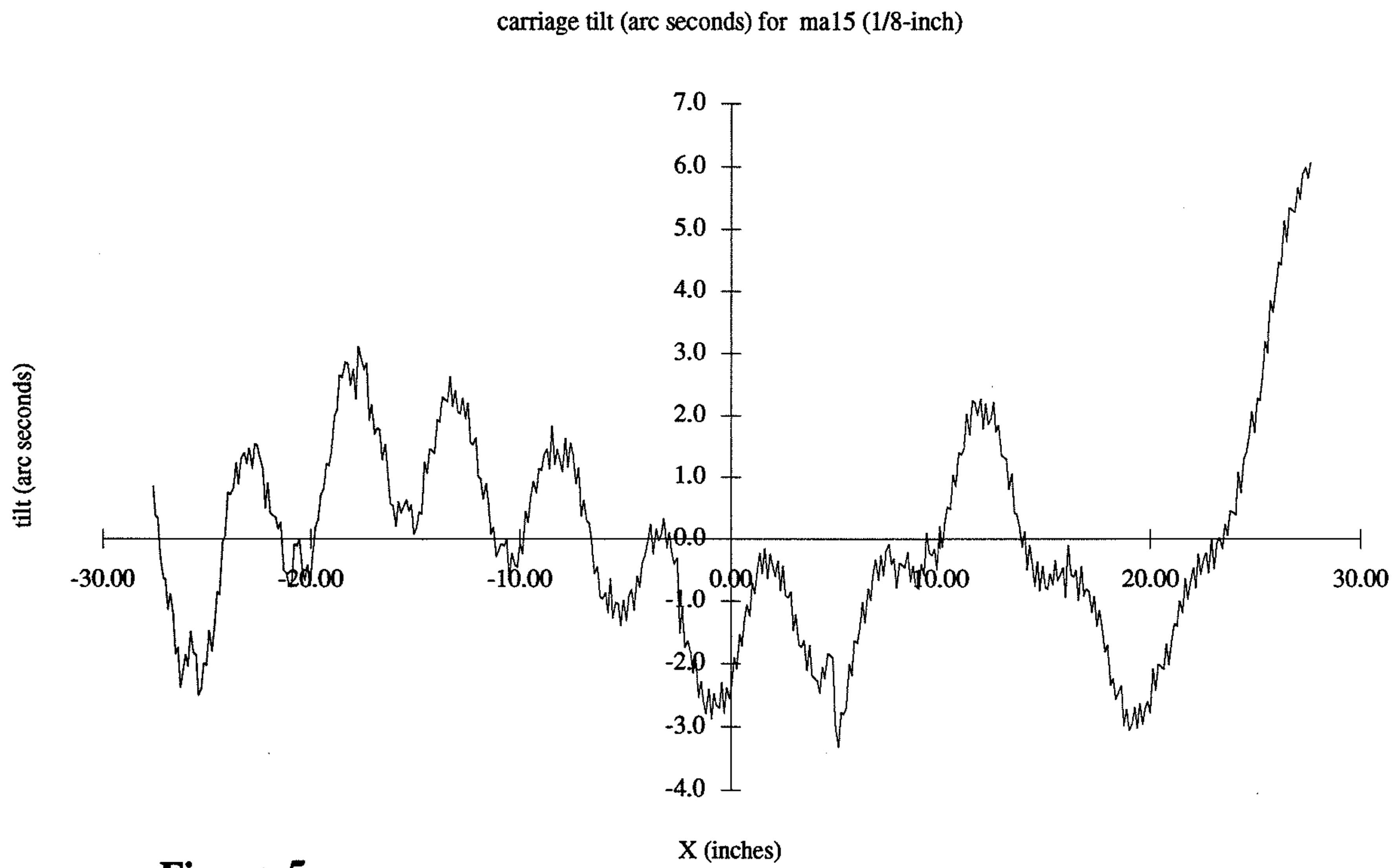


Figure 5

To calculate the correction for carriage-tilt we use the following:

P_{Θ}(ref_meas) = tilt * arm * slope

tilt = $\Theta_{carriage}(X)$

arm = vertical distance from the X-axis retroreflector to the probe tip.
This depends on the shape and vertical position of the part being measured, and it varies with X.

slope = the surface slope of the part being measured.
This depends on the shape of the part being measured, and it varies with X.

The carriage-tilt correction profile for the Keck 2 f/15 secondary mirror is shown in Figure 6. The largest effect is at the outer edge of the mirror where the carriage tilt, the lever arm, and the part slope all happen to be high. The maximum error is about 15 micro-inches (=0.38 microns). Note that the correction is not symmetric about X = 0, and thus leads to an asymmetry in the measured, uncorrected, profiles.

We also note that implementation of the overhead flat option will have no effect on this carriage-tilt correction.

carr-tilt-error ma15 ch

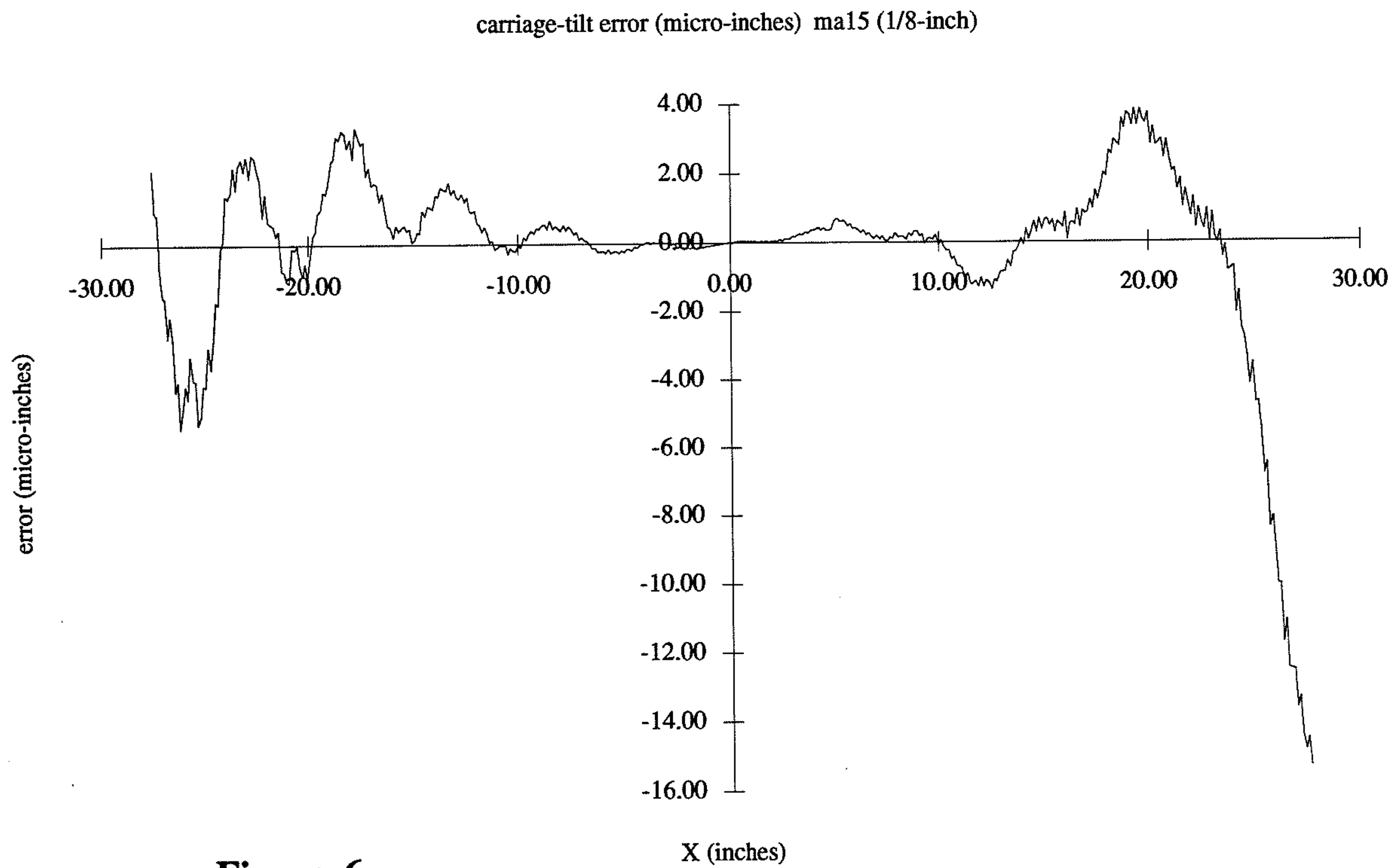


Figure 6

Errors in the Corrections for Carriage-Height and Carriage-Tilt

Errors in the measurement of the Carriage-Height correction lead to errors in P_Z(ref_meas). Based on the repeatability of the six measurements above,

δP_Z(ref_meas) = 1.27 / $\sqrt{N}$ micro-inches, where N is the number of full repeat measurements. This includes both the statistical errors and the effect of temperature variations.

Errors in the measurement of the Carriage-Tilt correction lead to errors in P_Θ(ref_meas). The error is dominated by the measurement of the carriage tilt. For the data set taken for the Keck 2 f/15 secondary, we estimate δtilt is about 0.5 arc seconds. Then

δP_Θ(ref_meas) = δtilt * arm * slope.

This varies with the position along the part. For measurements of the Keck 2 f/15 secondary, the error at the outer edge of the mirror (maximum arm and slope) is about 1.3 micro-inches.

9. Errors in Stitching

For many small optics we use a reference surface with a diameter that is equal or larger than the optic being measured. Thus the determination of the beam shape, P(beam) requires no stitching. For this case systematic variations in P(ref_meas) are dominated by the bridge curvature and lead to an error in the absolute radius of curvature of the beam shape and optic. The size of this effect is typically much smaller than the tolerance on the absolute radius of curvature.

For larger diameter optics, *e.g.* the Keck secondary, the effect is more serious. Stitching separate profiles is required to determine P(beam). Stitching removes the relative overlap-region piston and tilt and averages the profiles there. If the separate components were measured at different rail temperatures, then systematic errors are introduced in the stitching. Since we know the rail temperature for each measurement and we know the relation between rail temperature and bridge curvature, we can, in principle, remove this source of error. These corrections have not been made in the past or current work. We recommend that in the future each profile be corrected to a standard temperature.

In Appendix 9 we calculate the effect of statistical errors on the stitched profile. If the statistical error on a point in the profile is $\delta z_i = \sigma$, then averaging two fully-overlapping profiles will give $\delta z_i = \sigma/\sqrt{2}$. If the two profiles are not overlapping, and displaced as for the 40-inch flat references for the Keck secondary, then $\delta z_i = 1.2\ \sigma/\sqrt{2}$. We have not calculated explicitly the case of three profiles. Using the above we estimate the error on the stitched P(ref_meas) is

δz_i (statistical) ~ 1.2 $\sigma/\sqrt{3}$.

From Section 5 σ = 0.56 micro-inches = 0.014 microns

$\Rightarrow$ δz_i (statistical) ~ 0.39 micro-inches = 0.010 microns

Using the same method for the 20-inch flat gives

$\Rightarrow$ δz_i (statistical) ~ 0.37 micro-inches = 0.009 microns

The systematic error induced by variations of bridge curvature are also calculated in Appendix 9. Again we calculate for the Keck secondary and 40-inch flat reference. Using the variations in rail temperature observed during a beam shape measurement (0.3 ^{0}F rms), we estimate the error on the stitched P(ref_meas)

δz_i (systematic) ~ 2.2 micro-inches = 0.055 microns

10. Summary and Recommendations

Summary

The measurement process can be described by the following relations between profiles.

P(optic) = P(meas) – P(beam)

where P(beam) = Stitch[P(ref_meas) – P(ref_interfer)]

For large diameter optics P(beam) is determined by stitching together profiles of the reference optic measured at different positions along the bridge.

The surface of the optic is determined by combining multiple profiles P(optic).

Errors in a single profile measurement: P(meas) and P(ref_meas)

	X (μ-inches)	X (microns)	Z (μ-inches)	Z (microns)
statistical errors (rms)	0.44	0.011	0.56	0.014

systematic errors are dominated by the thermally-induced bridge curvature

$\delta k = -130$ microns / ^{O}F

Assuming changes from day to day of 10 ^{O}F gives errors in the absolute radius of curvature = 0.0013 meters.

Errors in reference surfaces P(ref_interfer)

	span (inches)	sag ± error in sag (micro-inches)	surface rms (micro-inches)	surface rms (microns)
20-inch flat	20	2.5 ± 0.3	0.1	0.003
40-inch flat	40	42.5 ± 2.5	< 2.4	< 0.060

	span (inches)	sag ± error in sag (inches)	surface rms (micro-inches)	surface rms (microns)
22-inch sphere	20	0.33800 ± 0.00001	< 0.6	< 0.005

Current data analysis software assumes the reference profile has **only** a sag error.

Errors in beam shape P(beam)

Unstitched. The errors depend on which flat is used for the reference.

20-inch flat

statistical error = 0.014 microns

systematic error = error in absolute radius of curvature = 0.0013 meters

40-inch flat

statistical error = 0.060 microns

systematic error = error in absolute radius of curvature = 0.0013 meters

Stitched.

Assuming the 40-inch flat is the reference optic

rms statistical error = 0.39 micro-inches = 0.010 microns

systematic in absolute radius of curvature = 0.0013 meters

rms systematic error from uncorrected curvature variations in stitching

= 2.2 micro-inches = 0.055 microns

combining the two rms errors in quadrature gives
rms error in P(beam) = 2.2 micro-inches = 0.056 microns

Errors in optic profile P(optic)

Combining the errors of P(meas) with those of P(beam) gives
rms error in P(optic) = 2.8 micro-inches = 0.070 microns
error in absolute radius of curvature = 0.0013 meters

Errors in optic surface

azimuthal symmetry

Our standard mode of using the profilometer and the standard manner of running the Curvmon program both assume that the measured surface is azimuthally symmetric.

In our standard mode of using the profilometer we measure a set of diameters at different angles and then remove piston and tilt from each profile since the rotation itself will introduce physical tilts of the optic.

In the standard mode of using Curvmon, the cubic term in each profile is removed and the cubic terms from all profiles are then used to calculate a global decenter (t_2 and t_3 of Appendix 7). The residuals from this fit are indicated in the standard output, but the residual cubic is not added back into the displayed profile errors.

As a result of both these, our output does not give a true picture of surfaces that vary azimuthally. Harland Epps has run test cases of non-azimuthally symmetric surfaces. As expected, local surface bumps cause artifacts in the output on the opposite side of the mirror. These are of order 1/3 to 1/2 of the original bump. Thus variations in the profiles should not be believed at the level of the azimuthal variations. However, the output does provide two indications of the presence of azimuthal variations, and **these warnings need to be heeded.**

1) **The residual profile plots can show azimuthal variations.**
2) **The rms residual from the fit to decenters, " RMS(F-C) " will be large if there are azimuthal variations.**

average of profiles

To determine the final values of the radius of curvature and conic constant we average multiple profiles. The resulting random errors are

$$\delta k = \frac{2k^2}{a^2} \frac{\sigma 4\sqrt{5}}{\sqrt{N}\sqrt{N_p}} \qquad \delta K = \frac{8k^3}{a^4} \frac{\sigma 48}{\sqrt{N}\sqrt{N_p}}$$

where is the rms error on each point, N is the number of points in a profile, and N_p is the number of profiles.

For the Keck secondary $\sigma = 0.070$ microns, N = 112, $N_p = 15$

(a = 0.723 meters, k = –4.738 meters)

yields rms errors δk = 1.31 microns δK = 0.00026

The error in the absolute radius of curvature is 0.0013 meters

off-axis distance

The position of the optical center with respect the mechanical center is addressed in Appendix 7. All points on all profiles lead to the fitted values of the x,y coordinates of the optical center, so the final statistical error on these values is small. For the Keck 2 f/15 secondary a set of five measurements (400 points / diameter, and 15 diameters) gave $\Delta x \sim \Delta y = \sim 0.0012$ inches.

Warning: The effects of the errors and sensitivities on the final surface measurement will be different for each optic, and they need to be re-considered separately for each optic.

Recommendations

We have regularly observed non-repeatable structure in the measured profiles. The exact origin of these is not determined; perhaps dust on the mirror, erratic floatation of the probe, electronic noise, etc. For the final measurements of an optic, where the highest precision and accuracy are required, we strongly recommend that multiple measurements of each profile be made. Only comparison and selection from multiple scans will ensure the achievement of the errors described in this report.

The largest correction made to the data is for carriage height variations along the beam. Additionally the quadratic component of this profile varies with temperature. This large source of uncertainty in the measured profiles is eliminated by using an overhead flat. We strongly recommend this option be fabricated, installed, and used.

Throughout this study we have seen no evidence for effects due to the probe shaft tilting in its air bearing. This effect undoubtedly exists at some level. More precise measurements or measurements of more steeply sloped optics may require investigation of this effect.

References

ANORAD Data Acquisition and Control System, (Dean Tucker, January 1993)

ANORAD Data Reduction Software (Steve Allen, September 1992)

Analysis of a Profilometer Repeatability Test (Terry Mast etal., April 1993)

The Effect of the Stage-1 Profilometer Upgrade (Terry Mast etal., March 1994)

Handbook of Formulas for Stress and Strain (Griffel, 1966)

Marks' Standard Handbook for Mechanical Engineers (Baumeister, Avallone, and Baumeister, 8th edition)

Timoshenko (J. O. S. A. volume 11, p 233 , 1925)

J. C. Owens (Applied Optics 6, No. 1, pp 51-59 (1967) (Eqns 29 - 31)

Appendices

Appendix 1. Tables of Mechanical Properties

Table 1. Materials Properties

	density	elastic modulus	thermal coefficient	specific heat	thermal conductivity
(units)	(g/cm^3)	(10^{10} N/m^2)	(10^{-6} /^oC)	(cal/(g^oC))	(cal /(sec ^oC cm))
Ruby	3.99	41	5	0.18	0.06
Zerodur	2.53	9	0.1	0.20	0.004
Steel	7.8	21	12	0.11	0.11
Granite	2.6	7	8	0.19	0.005
Aluminum	2.7	7	25	0.21	0.48
Pyrex	2.25	6.4	3.2	0.20	0.0024

Table 2. Profilometer Mechanical Parameters

Barry Mounts
- four mounts each 18 inches diameter and 2 inches high
- pressure 20 to 60 psi , mean = 40 psi

Steel Frame
- size 21 inches x 48 inches x 144 inches
- about 1/2 inch thick plate with diagonal cross beams
- approximate weight (~2.3 ft^3 of steel) = 1100 pounds = 500 kg

Granite Table
- size 12.5 inches x 48 inches x 144 inches
- height of surface above floor = 38 inches
- weight = 8,100 pounds = 3700 kg

Granite Bridge Support Blocks
- size height 13 inches x base 12 x 12 inches
- free distance between blocks 60 inches
- weight = 175 pounds / block = 80 kg / block

Granite Bridge
- size 84 inches x 12 inches x 12 inches
- weight = 1130 pounds = 515 kg

Linear Motor
- length 74 inches centered on the 84 inches of granite bridge
- winding spacing 0.30 inches

Carriage
- size 12 x 12 inches
- weight ~ 100 pounds
- air bearing gap 50 to 200 micro-inches
- air bearing area 3 pads, each 1.125 inches x 3.5 inches
- location of pads two front pads separated by 8 inches, back 5 inches begind

Probe Shaft
- Zerodur length 7.4 inches, square 0.435 x 0.435 inches
- part of length (0.8 inches) is round with 0.319 inch diameter
- weight = 55 grams

Probe Support
- float O. D. = 2 11/16 inches I.D. = 1 3/16 inches

height = 3/8 inches ⇒ volume = 1.71 $inches^3$
float fluid FC40 Flourinert specific gravity = 1.9
load on mirror with tip for f/15 = ~ 1.0 grams
adjustment system to change probe shaft angle
(two 10-80 screws spaced 4 and 1 10/16 inches)
free fall speed 0.46 ± 0.03 inches per second

Probe Tip
nominal radius 1 mm
material Ruby (Renishaw PS8R with a 10 mm shaft)
weight of ruby ball 0.017 grams
length of steel coupling ~6 mm

Table 3. Examples of Optics

f/15
size 1.445 m dia , 0.157 m thick at center, k = -4.738, sag = 0.055 m
right cyclindrical section 1.445 dia x 0.102 h = 0.167 m^3
average height = sag/2 1.445 dia x 0.0275 m = 0.045 m^3
volume = 0.212 m^3
weight = 536 kg = 1,180 pounds
aluminum support height = 5 inches high

20-inch flat
size 12.5 x 16.5 x 3 inches
volume = 0.010 m^3
weight = 25.7 kg = 56 pounds
high aluminum support height = 6.5 inches

40-inch flat
size 40 inch diameter x 7 inches
volume = 0.144 m^3
weight = 365 kg = 802 pounds
aluminum support height = 3.5 inches

22-inch sphere
size 22 inch diameter x 5.125 inch thick
volume = 0.032m^3
weight = 81 kg = 178 pounds

Appendix 2. Calculations of Sensitivities to Loads

Self-Weight Load on the bridge

Assume the bridge is a simply-supported beam.
The maximum deflection is

$$y_{max} = \frac{-5}{384}\frac{WL^3}{EI} \text{ (Griffel, page 16)}$$

L = mean distance between supports (72 inches)
W (betwen the suports) = 970 pounds
$I = ba^3/12 = 1728 \text{ inches}^4$

$$y_{max} \sim -270 \text{ micro-inches} \sim -6.8 \text{ microns}$$

The slope at the supports is

$$\theta = \frac{1}{24}\frac{WL^2}{EI} = 12 \text{ micro-radians}$$

Carriage Load on the Bridge

For a point load in the center of a simply supported beam

$$y_{max} = \frac{-1}{48}\frac{WL^3}{EI} \qquad \text{(Griffel, page 17)}$$

where L is the distance between supports (72 inches)
W = ~ 100 pounds $I = ba^3/12 = 1728 \text{ inches}^4$

$$y_{max} \sim -45 \text{ micro-inches} \sim -1.1 \text{ microns}$$

resonant frequency: $\omega^2 = g/y_{max} \Rightarrow f = 470 \text{ Hz}$

The slope at supports $\theta = \frac{WL^2}{16EI} = 1.9$ micro-radians

For a simply-supported beam of length L, with a point load W at a distance "a" from one end, and "b" from the other $a + b = L$. The deflection at any position x is

$$y = \frac{-Wa(L-x)}{6EIL}\left[2Lb - b^2 - (L-x)^2\right] \quad a < x < L$$

Since the probe is at the position of the load, $x = a$

$$y = \frac{-W}{6EIL}(L-b)^2\, 2\, b^2 \qquad 0 < b < L/2$$

or

$$y = \frac{-WL^3}{6EI}(1 - b/L)^2\, 2\, (b/L)^2$$

Using the above values we have

$y = 8.8\,(1 - b/L)^2\, 2\, (b/L)^2$ microns $0 < b < L/2$
This has quadratic, cubic, and quartic terms.

This is one contribution to the "beam shape." It is removed in any measurement by subtraction of the beam shape.

Carriage Load on the Air Bearings

The stiffness k ~ carriage weight / bearing thickness.

$$k \sim 100 \text{ pounds} / 100 \text{ micro-inches} \sim 10^6 \text{ pounds / inch}$$

This gives a resonant frequency: $\omega^2 = k/m \quad \Rightarrow \quad f = 310$ Hz

Loads on the Granite Table

The load on the granite table changes depending on the optic being measured and its positon on the table.

$$I = \frac{ba^3}{12} = 7813 \text{ inches}^4$$

Distributed Load

Consider a simply–supported beam of length L, with a uniformly distributed load W of length e, spanning the interval a to b ($b = a + e$).

Following Griffel (page 24) Let $d \equiv L - (1/2)(a + b)$.

The forces at the support points are $R_1 = (W/L)(d)$ and $R_2 = (W/2L)(a + b)$.

$$\text{Let } T \equiv \frac{8d^3}{L} - \frac{2be^2}{L} + \frac{e^3}{L} + 2e^2$$

$$f(x) \equiv \frac{8d}{L}(x^3 - L^2x) + Tx$$

The deflection y is

$$y_1 = \frac{W}{48EI} f(x) \qquad (0<x<a)$$

$$y_2 = \frac{W}{48EI} \left[f(x) - 2\frac{(x-a)^4}{e} \right] \qquad (a<x<b)$$

$$y_3 = \frac{W}{48EI} [f(x) - 2xe^2 - (2x - (a + b))^3 + e^2(2b - e)] \quad (b<x<L)$$

The end slopes are

$$\theta_1 = \frac{W}{48EI} [-8dL + T]$$

$$\theta_2 = \frac{W}{48EI} [24 dL + T - 24d^2 - 2e^2]$$

Consider only the heavy pieces in Table 3; the f/15 secondary and 40-inch flat.

The f/15 is positioned at the center of the bridge.
The 40-inch flat is positioned at the left, center, or the right of the gap.
The center of the bridge is 14 inches from the left support.

$L = 82$ inches $I = 7813$ inches^4

$$\frac{1}{48EI} = 2.67 \times 10^{-13} \text{ pounds}^{-1} \text{ inches}^{-2}$$

	e	a	b	d
f/15	57	-12.4		
40-inch flat at left	40	-16		
40-inch flat at right	40	4	44	58

The f/15 and 40-inch-on-the-left are over the support.

The 40-inch on the right is between the supports and will give the largest effect.

Using the above values yields

$$\frac{W}{48EI} = 2.1 \times 10^{-10} \text{ inches}^{-2}$$

$$T = 2.13 \times 10^{4} \text{ inches}^{2}$$

$$f(x) = 5.66\, x^{3} - 1.68 \times 10^{4}\, x$$

y_2 (x=a) = −14 micro-inches = −0.36 microns

y_2 (x=b) = −81 micro-inches = −2.05 microns

θ_1 = −3.5 micro-radians

θ_2 = 0.5 micro-radians

Profilometer on Barry Mounts

$$\frac{\delta P}{P} = \frac{-\delta V}{V} = \frac{-\delta h}{h} \quad \Rightarrow \qquad k = \frac{\delta PA}{\delta h} = \frac{PA}{h}$$

k = (40 psi) ($\pi\, 9^2$ inches^2)/(2 inches) = 5,090 pounds/inch

m = 8100 + 350 + 1130 + 1120 = 10,700 pounds

ω^2 = gk/m $\Rightarrow$ f = 2.2 Hz

Vertical Load of the Probe Shaft

For a sphere with radius r acting with force P on a plane (Baumeister *Marks'* 5-51)

Contact area radius = R

$R^3 = 0.68\, P\, r\, (c_1 + c_2)$ where $c_i = 1/E_i$

Maximum Contact Pressure (at center of contact) = S_{max}

$S_{max} = 1.5\, P / (\pi R^2)$

Total Deformation of the Two Surfaces = Y

$Y^3 = 0.46\, P^2 (c_1 + c_2)^2 / r$

Assume P = 1.0 grams

R = 180 micro-inches = 4.5 microns

S_{max} = 3.2×10^4 psi

Y = 0.8 micro-inches = 0.020 microns

Horizontal Loads on the Probe Shaft

A horizontasl load will be applied to the probe tip when measuring a tilt surface. For the Keck secondary the maximum slope is 0.15 radiansl, resulting in lateral force of about 0.15 grams. This will move the probe tip laterally. If the effect is linear, then the resulting change in height will be δz will be proportional to | r/k |, resulting in an error in the absolute radius of curvature.

The size of the effect could be measured with a autocollimator.

Appendix 3. Calculations of Sensitivities to Temperature Changes

Temperature variations induce changes in

1. the tilt and piston of the optic on the table
2. the tilt and piston of the bridge
3. the curvature of the bridge
4. steel and air in the components defining the X-axis counts
5. steel and air in the components defining the Z-axis counts

We estimate the size of each of these effects using the material properties listed in Table 1 of Appendix 1.

Piston and Tilt of the optic on the table

The f/15 mirror is supported by an aluminum base with thickness 5 inches.

Piston = 69 micro-inches / ^{o}F = 1.8 microns / ^{o}F

Piston and Tilt of the bridge

If the temperature of both support blocks change by 1 ^{o}F, then
Piston = 58 micro-inches = 1.5 microns

If the temperature of one support block is changed by 1 ^{o}F, then
Tilt = 0.8 micro-radians

Curvature of the bridge

The metal base of the linear motor is coupled along its length to the granite beam. Temperature changes induce a "bi-metal" curvature change in the granite beam. Timoshenko (J. O. S. A. volume 11, p233 , 1925) gives the formula for the radius of curvature (ρ) induced by a temperatue change.

$$\frac{1}{\rho} = \frac{(\alpha_2 - \alpha_1)\,\Delta T}{\frac{h}{2} + \frac{2(E_1 I_1 + E_2 I_2)}{h}\left(\frac{1}{b_1 E_1 a_1} + \frac{1}{b_2 E_2 a_2}\right)}$$

where α_i are the coefficients of thermal expansion

ΔT = the temperature change
E_i = elastic moduli, a_i = thicknesses, b_i = widths
I_i = moments of intertia

$h = a_1 + a_2$ $\Delta\alpha = \alpha_1 - \alpha_2$ R = 30 inches

	E	I	a	b	α	EI	1/bEa
	(10^{+6} psi)	(inches^4)	(inches)	(inches)	(10^6/^{o}F)	(10^{+6} pi^2)	(10^{+6} p^{-1})
Al	10	0.130	0.75	1.75	12.5	1.30	0.0762
Steel	30	0.018	0.375	4	6.5	0.54	0.0222
Gr	10	1728	12	12	3.5	17,280	0.000694

Consider different pairs for #1 and #2

#1	#2	h	$\Delta\alpha$	$sag = \frac{R^2}{2\rho}$
aluminum	steel	1.125	6×10^{-6} /^{o}F	3061 micro-inches/^{o}F
steel	granite	12.375	3×10^{-6} /^{o}F	18.5 micro-inches/^{o}F
aluminum	granite	12.75	9×10^{-6} /^{o}F	18.8 micro-inches/^{o}F

For either metal + granite , to a good approximation

$$\frac{1}{\rho} = \frac{\Delta\alpha\ \Delta T\ h\ b_{Me} E_{Me} a_{Me}}{2(E_{Gr} I_{Gr})}$$

By coincidence $\Delta\alpha\ E_{Me}$ is the same for aluminum and steel, so the two-layer metal can be considered as one, and the bi-metallic formula applies with the sum of the ba values, or the sum of the two sags.

sag (over 60 inches) = 37.3 micro-inches/^{o}F.

Steel and Air in the X-axis

A change in the steel between scans shifts the X-counts and has no effect on the results. A change in the air temperature between scans is corrected for in the conversion from counts to inches. Changes in either of these during a scan is not corrected, and both lead to a scaling of the X counts during the scan.

$X \Rightarrow X(1 + X\Delta)$.

where $\Delta = f/(2a)$ and 2a is the full range

f = the fractional change over the full X range, $0 < f < 1$

For

$$z = \frac{(X + X^2\Delta)^2}{2k} = \frac{X^2}{2k} - \frac{X^3\Delta}{k}$$

This effect generates a cubic in the profile.

$$\delta a_3 = \Delta \frac{a^3}{k}$$

Steel

First assume a vacuum and consider the effect of temeprature changes on the steel. The servo holds the x-counts fixed. The only steel distance contributing to the X counts is the reference arm of the interferometer, about 1 inch between the interferometer and the retro-reflector. As the temperature increases, the reference arm length increases, and the X counts decrease. The measured X is smaller than the actual X. We assume the coefficent is 12×10^{-6} /^{o}C = 6.7×10^{-6}/^{o}F.

If the temperature changes **during** a profile measurement, then there will be a scaling of X. For the 1-inch reference arm length , the change at the end of the scan is $2 \times 6.7 \times 10^{-6}/^{o}F = 1.3 \times 10^{-5}$ inches / ^{o}F.
For the Keck secondary 2a = 55 inches

Thus $f = 2.4 \times 10^{-7} \ / \ ^{o}F$

Air

Now assume there is no change in the steel and the air temperature changes.

Index of refraction of air

$$\lambda_{air} = \frac{\lambda_{vac}}{n} \qquad n = 1 + 2.73 \times 10^{-4} \left(\frac{288}{T}\right)\left(\frac{P}{1000}\right)$$

If $T = 300$, $P = 1000$, $n - 1 = 2.62 \times 10^{-4}$

$$\delta n = (n-1)\frac{-\delta T}{T} \text{ (T in Kelvin)} \qquad \delta n = (n-1)\frac{\delta P}{P} \text{ (P in millibars)}$$

Again X scales.

If the temperature increases, the index n decreases, λ_{air} increases and the X counts decreases. Again the measured X is smaller than the actual X.

$$\frac{\delta \text{ counts}}{\text{counts}} = \frac{\delta n}{n} = 2.62 \times 10^{-4}\left(\frac{-\delta T}{T} + \frac{\delta P}{P}\right)$$

The conversion from counts to meters (Allen) corrects the conversion factor using TEMP0, the air temperature for the scan. We consider here the effect of the temperature change **during** the scan. This is a changing scale factor

$$f = 2.62 \times 10^{-4}\left(\frac{-\delta T}{T} + \frac{\delta P}{P}\right)$$

If $T = 300$ and $dP = 0$, then $f = 8.7 \times 10^{-7} \ / \ ^{o}F$

Example:

Assume $\delta T = 1\ ^{o}F$ **during** a scan of the Keck secondary.

The total for steel and air gives $f = 1.1 \times 10^{-6} \ / \ ^{o}F$.

$\delta a_3 = f \frac{a^2}{2k} = f$ (55,200 microns) = 0.061 microns / ^{o}F.

On Sept 20 the Keck secondary was measured with 15 profiles. The ambient temperature drifted linearly with time, and during the time of a single scan (7.4 minutes) the temperature changed by 0.063 ^{o}F. This implies

$\delta a_3 = 0.0038$ microns, a negligible amount.

Steel and Air in the Z-axis

Steel/Zerodur

The optical pathlengths are defined by the plane mirror above the interferometer (M1), the plane mirror at the top of the probe shaft (M2), and the reference arm retroreflector (R). Thermally-induced changes in the positions of these elements leads to errors in the Z counts.

The interferometer measures the distance (D) between the two plane mirrors (M1 and M2) relative to the 2 times the distance to the reference arm retro-reflector. A change in these distances between scans simply changes the Z values by a constant. A change in this distance during a measurement to first order adds a tilt to the profile. Both of these are removed in the data analysis and thus have no effect on the final results.

Similarly, changes in the length of the probe shaft or the coupling to the ruby ball will also add constant and linear terms to the profile.

If there are non-linear variations in these lengths, then those will affect the measured profile. We calculate the size of the constant and linear terms in order to estimate the likelihood of there being signficant non-linear variations.

R) The reference arm length is 2 x 1 inch. If the temperature increases, this distance increases, and the Z values decrease. Using 12×10^{-6} ^{o}C we get a change in the Z values of −13.3 micro-inches / ^{o}F.

M1) There is a distance of 2 inches between the carriage air bearing surface and the plane mirror M1. If the temperature increase, the distance increases, reducing the distance between M1 and M2, and giving a surface reading higher than the actual surface. Using 12×10^{-6} ^{o}C we get a change in the Z values of +13.3 micro-inches / ^{o}F.

M2) If the temperature increases, the Zerodur shaft and probe tip shaft lengths increase, raising M2, decreasing D, and giving a surface height reading that is higher than the actual surface. Using 12×10^{-6} ^{o}C for the probe tip shaft we get a change in the Z values of 0.25 (12×10^{-6} ^{o}C) 0.55 = +1.7 micro-inches / ^{o}F. Using 1×10^{-7} ^{o}C for the Zerodur shaft we get a change in the Z values of 7.4 (1×10^{-7} ^{o}C) 0.55 = +0.4 micro-inches / ^{o}F.

The sum of all these effects is +2.1 micro-inches / ^{o}F = 0.053 microns / ^{o}F.
During a diameter scan, a typical change in temperatue is 0.3 ^{o}F; giving a surface height error of 0.6 micro-inches = 0.016 microns. These are negligble and non-linear variations will be even smaller.

Air

The air pathlength is 2 inches in the reference arm and varies from 3 inches minimum to 5 inches maximum in the distance between M1 and M2. Thus the maximum air difference is 3 inches. A temperature change scales Z by the factor $(1 + \delta)$ where $\delta = 2.62 \times 10^{-4} (-(\delta T/T) + (\delta P/P))$. For 3 inches the change in z is 1.4 micro-inches = 0.037 microns. Since we expect these to be linear in time, the effect will be removed in the data analysis.

Appendix 4. Measurements of Thermal-Induced Bridge Curvature.

On 20 and 21 September we measured the Keck secondary on 15 different diameters. The temperature varied during the measurements, providing data on the variation of the beam curvature with temperature. The 15 profiles are at different angles and thus include the effect of surface variations in the secondary. After removing a linear temperature variation from the Curvmon-fitted radius of curvature, we see a cos2θ angular dependence with an amplitude of about 30 microns. We assumed this is due to astigmatism in the surface, and removed it from the Curvmon curvatures. These corrected curvatures are plotted versus rail and ambient temperature in Figure A4-1. The linear variation with rail temperature is clear. There is a larger scatter in the variation with ambient temperature, suggesting the effect is more tightly coupled to the rail temperature, as expected. The trend in the data gives $\delta k = -130$ microns / ^{o}F.

This corresponds to a quadratic change

$$\delta a_2 = \frac{-a^2}{2k^2}\delta k \quad -1.51 \text{ microns} / {}^{o}\text{F} \qquad \text{where } \delta z = \delta a_2 \left(\frac{x}{a}\right)^2$$

As the temperature increases, the bridge bows up, increasing the sag, and decreasing the measured radius of curvature.

There is a small shift between the two data sets. This may be due to a different granite bridge temperature (~ 0.5 degrees), which we expect to be constant during a measurement set, but could vary from day to day.

In April 1993 measurements of the 20-inch flat (Mast etal. 1993) showed a change in radius with the ambient temperature that agrees (to ~ 25%) with the above value. We believe this bridge curvature effect has been always present at this level.

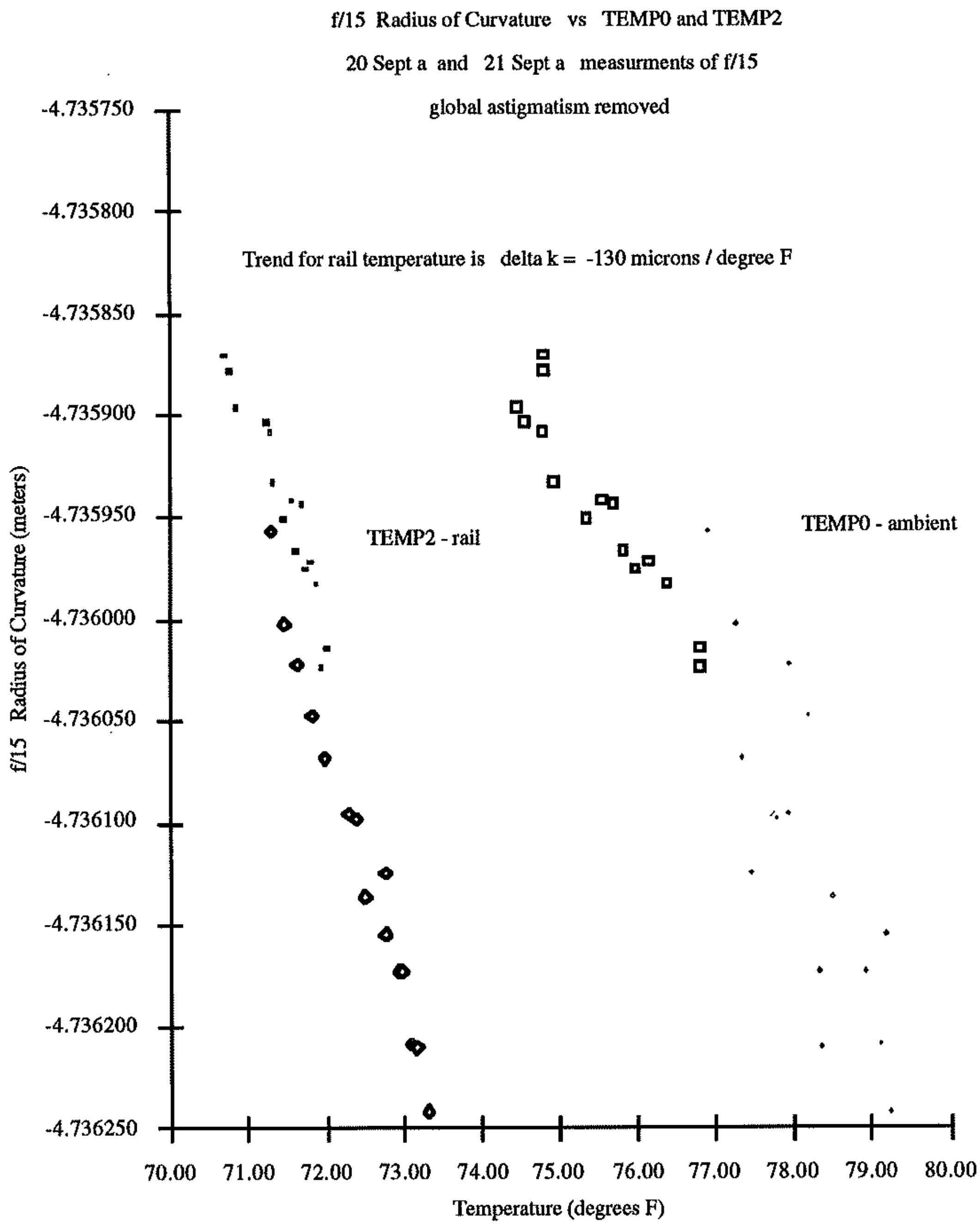


Figure A4-1. Measured Radius of Curvature versus Rail and Ambient Temperature.

Appendix 5. Conversion from Counts to Meters

The output of the profilometer is X and Z in counts. Each X-count is a wave / 60 and each Z-count is a wave / 120. To convert to meters

$$X_i \text{ (meters)} = X_i \text{ (counts)} \frac{\lambda_{air}}{60} \qquad Z_i \text{ (meters)} = Z_i \text{ (counts)} \frac{\lambda_{air}}{120}$$

$$\text{where} \qquad \lambda_{air} = \frac{\lambda_{vac}}{n} \qquad \lambda_{vac} = 632.99137 \text{ nm}$$

$$n = 1 + 2.729 \times 10^{-4} \left(\frac{288.15}{T}\right)\left(\frac{P}{1000}\right) \qquad [T] = {}^{0}\text{ Kelvin} \quad [P] = \text{millibar}$$

J. C. Owens (Applied Optics 6, No. 1, pp 51-59 (1967) (Eqns 29 - 31)

If $T = 300$, $P = 1000$, then $n - 1 = 2.62 \times 10^{-4}$

Consider errors in T and P , ΔT and ΔP, then

$$\frac{X'(\text{meters})}{X(\text{meters})} = \frac{Z'(\text{meters})}{Z(\text{meters})} = \frac{n}{n'} = 1 + 2.729 \times 10^{-4} \left(\frac{\Delta P}{1000} - \frac{\Delta T}{288.15}\right) \equiv 1 + \delta$$

So ΔT and ΔP lead to a scaling of x and z : $x' = x(1 + \delta)$ $\quad z' = z(1 + \delta)$

Scaling in x alone changes z(x). For small δ the scaled $z(x(1+\delta))$ is

$$z(x(1+\delta)) = a_{20}(1+2\delta)\left(\frac{x}{a}\right)^2 + a_{40}(1+4\delta)\left(\frac{x}{a}\right)^4 + a_{60}(1+6\delta)\left(\frac{x}{a}\right)^6 \ \ldots.$$

With both x and z scaling

$$z(x) = a_{20}(1+\delta)\left(\frac{x}{a}\right)^2 + a_{40}(1+3\delta)\left(\frac{x}{a}\right)^4 + a_{60}(1+5\delta)\left(\frac{x}{a}\right)^6 \ \ldots.$$

Consider some examples of surfaces recently made in our shop.

Let $\quad \Delta T = -11\ {}^{0}C \quad \Rightarrow \quad \delta = +10^{-5}$

or $\quad \Delta P = +37$ millibar $\Rightarrow \quad \delta = +10^{-5}$

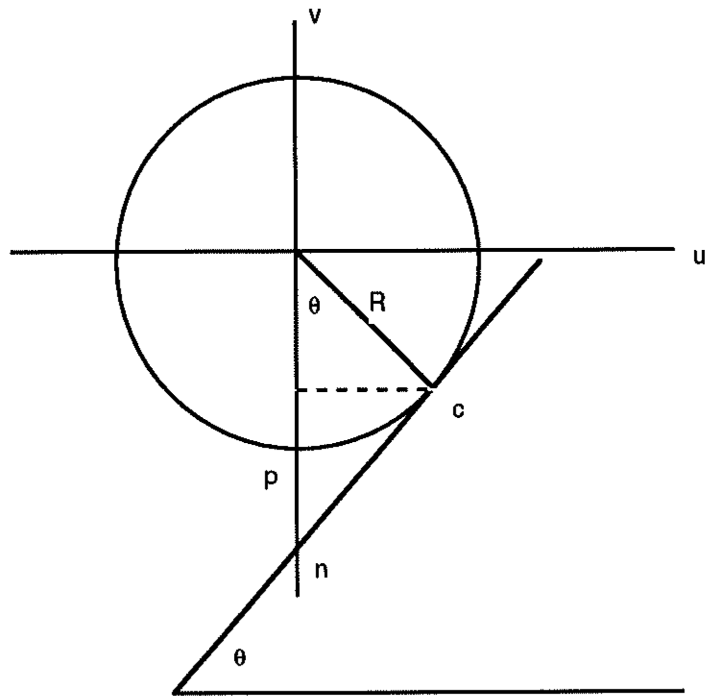
$$\Rightarrow \frac{\delta a_{20}}{a_{20}} = \delta = 1 \times 10^{-5}$$

		Keck f/15	KAST	NORIS	UMICH	DEEP1	DEEP8
a20	microns	-55164.444	15430.667	24313.921	2272.938	23605.885	49073.283
a40	microns	206.922	-410.694	13.150	1131.528	-5705.404	7749.486
a60	microns	-1.552	-251.653	-118.894	960.263	-1387.788	3144.180
a80	microns	0.015	-367.879	-298.948	-251.310	-1136.387	1270.875
δa20	microns	-0.552	0.154	0.243	0.023	0.236	0.491
δa40	microns	0.006	-0.012	0.000	0.034	-0.171	0.232
δa60	microns	0.000	-0.013	-0.006	0.048	-0.069	0.157
δa80	microns	0.000	-0.026	-0.021	-0.018	-0.080	0.089

Typical manufacturing tolerances on a_{20} (or the radius of curvature) call for 1 part in 10^4. The large errors assumed here ($\Delta T = -11\ {}^{0}C$ or $\Delta P = +37$ millibar) give an error about 10 times smaller than this typical manufacturing tolerance.

Appendix 6. Correction for Finite Probe-Tip Diameter

Consider a probe tip of radius R in contact at with a plane of slope $s = \tan\theta = \partial f(x) / \partial x$



The u,v coordinates of the points p, n, and c are as follows:

$$u_p = 0 \qquad v_p = -R$$

$$u_n = 0 \qquad v_n = R / \cos\theta = -R\,(1 + s^2)^{1/2}$$

$$u_c = R \sin\theta = R\,s / (1 + s^2)^{1/2} \qquad v_c = R\ \cos\theta = -R / (1 + s^2)^{1/2}$$

Then

$$u_p - u_c = -R\,s / (1 + s^2)^{1/2} \qquad v_p - v_c = -R + R / (1 + s^2)^{1/2}$$

These agree with Steve Allen's report page 6.

There are two options here: use point n or point c. Steve has chosen to use point c. An advantage of choosing point n is that the equal spacing of the data points is preserved.

Size and Effect of Correction

The surface is dominated by $a_{20} (\frac{r}{a})^2$, and the slope is dominated by $2a_{20} \frac{r}{a^2}$.

Thus $(1 + s^2)^{1/2} \sim 1 + \frac{1}{2} s^2 \sim 1 + 2\ (\frac{a_{20} r}{a^2})^2$

giving $v_n = -R\,(1 + s^2)^{1/2} \sim -R - 2R\,(\frac{a_{20}}{a^2})^2\ r^2$

⇒ There is a constant and a quadratic correction to the height.

The quadratic term leads to a shift in a_{20} of $\Delta a_{20} = 2R\,(\frac{a_{20}}{a})^2$.

The fractional shift is $\frac{\Delta a_{20}}{a_{20}} = \frac{2\,R\,a_{20}}{a^2}$.

Consider some examples of surfaces recently made in our shop and assume R = 1 mm.

		Keck f/15	KAST	NORIS	UMICH	DEEP1	DEEP8
a	meters	0.723	0.066	0.106	0.026	0.157	0.130
a20	microns	-55164.444	15430.667	24313.921	2272.938	23605.885	49073.283
a40	microns	206.922	-410.694	13.150	1131.528	-5705.404	7749.486
a60	microns	-1.552	-251.653	-118.894	960.263	-1387.788	3144.180
a80	microns	0.015	-367.879	-298.948	-251.310	-1136.387	1270.875
Δ20	microns	11.6	111.0	104.7	15.3	44.9	287.0
Δ20/a20	1	-2.11e-04	7.19e-03	4.30e-03	6.72e-03	1.90e-03	5.85e-03

Conclusion: It is essential to include the probe tip correction.

Errors in the Probe-tip Correction.

Errors in the correction arise from an error in the radius R or in the slope s.

δR: $$\frac{\delta\Delta a_{20}}{\Delta a_{20}} = \frac{\delta R}{R} .$$

If $\delta R = 0.010$ mm and $R = 1.0$ mm, then $\delta\Delta a_{20} = 0.01\ \Delta a_{20}$.

For the Keck secondary $\delta\Delta a_{20} = 0.116$ microns, and the resulting fractional error in radius of curvature is

$$\frac{\delta k}{k} = \frac{\delta a_{20}}{a_{20}} = 2.1 \times 10^{-6}$$

δs:

In practice we don't know s because we make the probe-tip correction before fitting to the conic. We consider here the option of approximating s by

1. Using the design conic, and
2. Approximating the design conic by only the first term

$$z = \frac{r^2}{2k_{design}} \qquad s = \frac{-r}{k_{design}}$$

This will induce an error in s from two sources.

1. There is an error in k. $\Delta k = k_{real} - k_{design}$ $\Delta s_1 = \frac{r}{k^2}\Delta k$

2. We neglect higher terms, the largest is $\Delta s_2 = \frac{(K+1)r^3}{2k^3}$

Which error is larger?

$$\frac{\Delta s_2}{\Delta s_1} = \frac{(K+1)r^2}{2k\ \Delta k} \quad \text{and maximum} \quad \frac{\Delta s_2}{\Delta s_1} = \frac{(K+1)a^2}{2k^2\ (\Delta k/k)}$$

For the Keck secondary $\frac{(K+1)a^2}{2k^2} = 0.0075$, $\frac{\Delta k}{k} \sim 10^{-3}$ to 10^{-4}

$\Rightarrow$ Δs_2 is largest (Our neglecting the higher terms.)

$$\text{Thus } \delta v_n = -R\,(1 - \frac{r^2}{2k^2})\,(\frac{r}{k})\,(\frac{(K+1)r^3}{2k^3}) \sim (\frac{-R(K+1)a^4}{2k^4})\,(\frac{r}{a})^4$$

This is an error in a_4 and leads to an error in the conic constant K.

$$\frac{\delta K}{K+1} = \frac{\delta a^4}{a^4} \qquad\qquad \delta K = (K+1)\,\frac{R}{4k}$$

Example

For the Keck secondary $\delta K = 3.4 \times 10^{-5}$. The tolerance for the Keck secondary is $\delta K < 0.0008$.

Conclusion

Using $s = \frac{r}{k_{design}}$ is a completely adequate method for calculating the slope for the probe-tip correction.

Appendix 7. Cubic Terms and the Off-Axis Distance

There are multiple descriptions of a "cubic" term in a profile.

For a pure cubic we use

$$z = a_3 \left(\frac{x}{a}\right)^3 \quad \text{where } -a < x < a$$

1) a best-fit tilt = $a_1\, x$ where $a_1 = -(3/5)a_3$

2) an rms height = $a_3 / \sqrt{7}$

For a cubic with the best-fit tilt removed

$$z_{tr} = a_3 \left[\left(\frac{x}{a}\right)^3 - \frac{3x}{5a} \right]$$

1) the maximum internal amplitude = $\frac{2}{5\sqrt{5}}\, a_3$

2) the rms height = $4\, a_3 / 175$

There are three sources of cubic terms in the profile:

i) non-perpendicularity between the profilometer X and Z axes

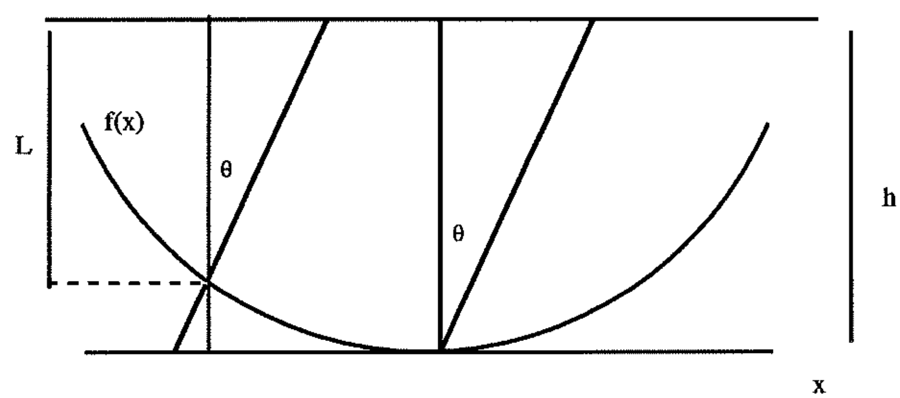
i) surface variations that lead to cubic terms in a profile (for example pure Zernike terms $C_{3\pm1}$, $C_{3\pm3}$ or higher order spatial frequencies with $C_{3\pm1}$, $C_{3\pm3}$ components)

iii) displacement of the optical center from the mechanical center.

We first consider the effect of the non-perpendicularity of the axes. Then we address the general problem of cubics in the measured profiles.

Non-Perpendicularity of X and Z axes

Consider a probe shaft at a small angle θ to the x axis.
Assume the true surface is has height $z = f(x)$.



If $\theta = 0$, then the measured shaft extension = $L(x) = h - f(x)$.

If $\theta \neq 0$, then the measured shaft extension = $L(x) = (h / \cos\theta) - (f(x-\Delta) / \cos\theta)$

$$\text{where } \tan\theta = \frac{\Delta}{f(x)} \quad \Rightarrow \quad \Delta \cong \theta \;\; f(x)$$

So there are two effects:
1) a lengthening of the distances by $1/\cos\theta$
2) a shifting of the x-axis points

by an amount that depends on f(x).

$$L(x) = \frac{1}{\cos\theta} [h - f(x-\Delta)]$$

$$\text{where} \quad f(x-\Delta) \cong f(x) - \Delta \frac{\partial f}{\partial x} \cong f(x) - \theta f(x) \frac{\partial f}{\partial x}$$

Let $f(x) = \frac{x^2}{2k} + \frac{x^4}{8k^3}$ (the first two terms in a sphere)

$$\text{Then} \quad f(x-\Delta) \cong f(x) - \theta \left(\frac{x^2}{2k} + \frac{x^4}{8k^3} \right) \left(\frac{x}{k} + \frac{x^3}{2k^3} \right)$$

$$\cong f(x) - \theta \left(\frac{x^3}{2k^2} \right)$$

Conclusion: A misalignment angle θ leads to a cubic term $a_3 = \frac{\theta a^3}{2k^2}$

Note:

The above is for a pure cubic term, x^3. If one measures instead a cubic shape with tilt removed, then

$z_{tr} = a_3 [(\frac{x}{a})^3 - \frac{3x}{5a}]$ and the maximum amplitude of the tilt-removed curve (read from the plot) $= \frac{2}{5\sqrt{5}} a_3 = 0.179 \ a_3$.

Example:

On 20 Sept 94 we measured a test sphere (k = 0.347 meters and a = 0.146 meters).

We measured a tilt-removed, maximum amplitude = 8 x 10^{-6} micro-inches = 0.254 microns. This yields a_3 = 1.14 microns and

$$\theta = a_3 \frac{2k^2}{a^3} = 18 \text{ arc seconds.}$$

For an off-axis distance $R = \frac{2k^3}{Ka^3} a_3$ (K.O. Report 91).

$$R = \frac{\theta k}{K} \quad \text{and} \quad \theta = \frac{RK}{k}$$

Measurements of the Keck secondary (a = 0.723 m, k = –4.738 m, K = –1.644)

20/21 Sep 94 gave ACONST = –0.0067, –0.0070, and –0.0051 inches
mean ACONST = 0.16 mm. stdev ACONST = 0.03mm

θ = 11 ± 2 arc seconds

Measurements of the LAGOS lens (a = 0.145 m, k = –1.47 m, K = –2.06)

ACONST = + 0.0096 inches (0.24 mm) implies θ = + 69 arc seconds.

Analysis of Cubic Terms and Off-axis Distance

For each profile P(u) (-1 < u < +1) at angle β we can fit to monomials

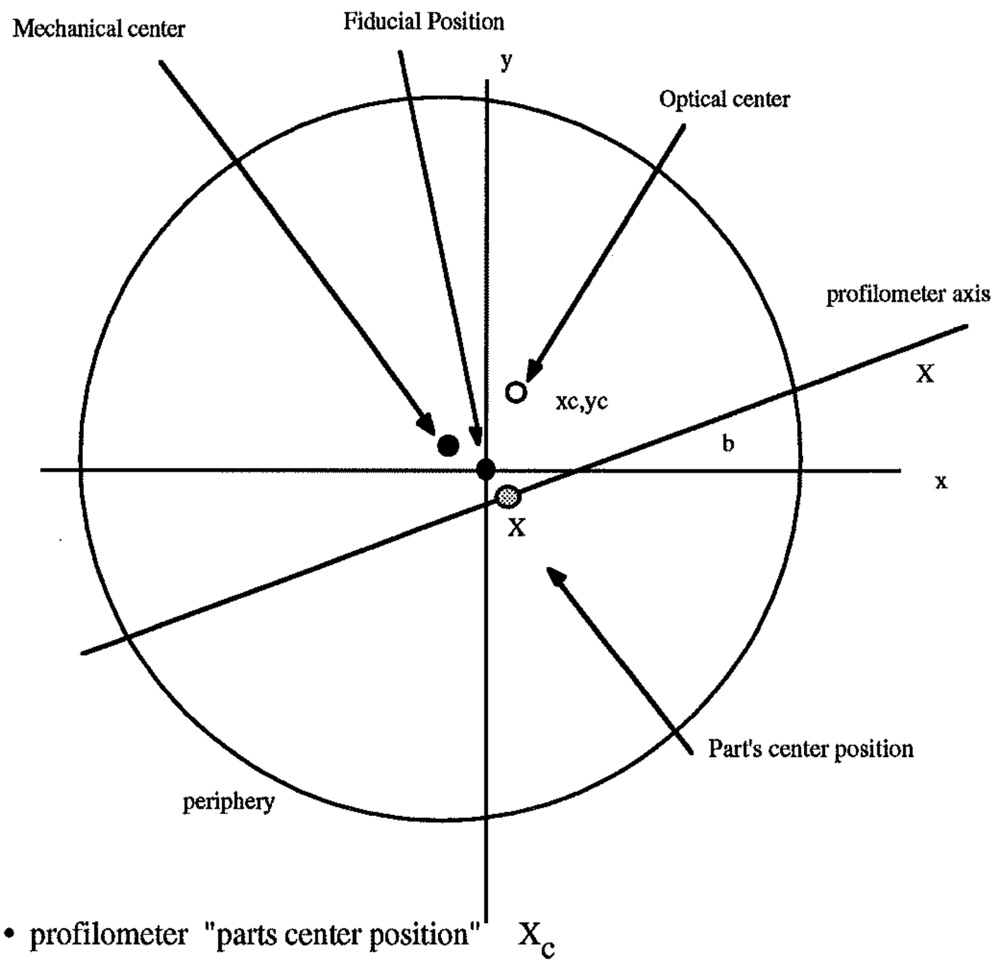
$$z = c_0 + c_1 u + c_2 u^2 + c_3 u^3 + \ldots \text{ to find the cubic coefficients } c_3$$

The values of c^{β}_3 for all β's can then be fit to the function

$$t_1 + t_2 \cos\beta + t_3 \sin\beta + \text{residuals}$$

The residuals and best fit parameters t_1^*, t_2^*, t_3^* are described in turn below.

In the chart below we define various centers and coordinates



- profilometer "parts center position" X_c

  This is a position **defined** by the operator when she uses the command `partctr` in the normal measuring process.
  (There is no error on this defined number.)

- "mechanical center" of blank =

  the mathematical average center of the periphery of the blank.
  (There is no error in this ideal position.)

- "fiducial position"

  $x = y = 0$, where x,y is the origin of the data-analysis-derived surface.

The fiducial is placed on the glass using runout on the edge.

The intention is to place the fiducal at the mechanical center.

This fiducial may be marked after each polish cycle.
It is marked by achieving zero runout from the edge.
Ideally it would be at the "mechancial center", but it will miss by some error.

- "optical center" of surface x_c, y_c

  Ideally it will be at the fiducial position, but in practice it will be displaced away from that position.
  Its position will change with each polishing cycle.
  The position must be calculated using the profilometer data.
  The error in the calculated position will depend on measurement errors,
  data analysis approximations,
  and errors in positioning the optic in the profilometer.

In the meauring process

1. A fiducial is placed on the glass, using a rotary stage.
   errors $\delta x = \delta y \sim 0.010$ inches
2. The carriage moves to X_c
   rms positioning error < 1 micron
   Compared to the other errors, this is negligible.
3. The part position is adjusted to visually place the probe tip on the fiducial.
   errors $\delta x = \delta y \sim 0.010$ inches

Residuals

The residuals result from random errors in aligning the mechanical center with the profilometer center and from higher order spatial frequency surface variations.

If the error is σ_X (meters), then the resulting rms scatter in c_3 is

$$\sigma_{a3} = \frac{Ka^3}{2k^3}\sigma_X$$

Constant term t_1^*

t_1^* arises from the non-perpendicularity of the X and Z profiometer axes.

We do not expect the non-perpendicularity to change from profile to profile.

Thus we associate the t_1 term (independent of β) with this effect.

$$\theta = \frac{2k^2}{a^3}t_1$$

Varying terms t_2^* and t_3^*

t_2^* and t_3^* arise from the displacement between the optical surface center and profilometer center, or from pure cubic terms in the surface, or from both.
Without further information we cannot assign them to one source or the other.

Keck Observatory Report No 91 describes the effect of being off-axis on a conic. An optical surface off-axis from the mechanical center by Δx and Δy, has the following values for the cubic terms.

$$z(\rho,\theta) \approx \frac{-a^3 K \Delta x}{2k^3} \rho^3 \cos\theta + \frac{-a^3 K \Delta y}{2k^3} \rho^3 \sin\theta$$

Giving

$$t_2^* = \frac{-a^3 K \Delta x}{2k^3} \qquad t_3^* = \frac{-a^3 K \Delta y}{2k^3}$$

Errors in the measurement of each profile lead to an error in the measurement of the $c\beta_3$ and then to errors in t_2^* and t_3^*. These lead to errors in Δx and Δx. Rather than propagate the errors through this chain, we cite an empriical error as an example. The five measurements of the Keck 2 f/15 secondary made on April 5, 6, 10, and 11 (with 15 diameters / measurement and 400 points / diameter), yielded an off-center amplitude $A = 0.0395 \pm 0.0012$ and angle $\theta = 142.9 \pm 1.7$ degrees ($\Delta x = A \cos\theta$, $\Delta y = A \sin\theta$).

Appendix 8. Carriage-Height and Carriage-Tilt Errors

The air-bearing surface of the granite bridge is not perfectly flat As the carriage moves in X, both it's height and tilt change by small amounts.

The variations of the height, $Z_{carriage}(X)$, we call the "carriage-height profile"

The variations of the tilt, $\Theta_{carriage}(X)$, we call the "carriage-tilt profile."

Note: The previous terminology "beam.shape" refers to the carriage-height profile.

Carriage-Height Error

The carriage-height profile is measured using a known reference flat. This has been a standard part of the measurement and analysis procedures and results in the file called "beam.shape" which is subtracted from the measured heights of the part.

We report here an example using the 20-inch flat measured at five positions along the bridge. We measured the carriage-height profile nine times with 1/8-inch spacing at the same sample points used to measured the Keck 2 secondary.

	file	ΔT (^{o}F)	rms res (micro-inches)	mean T (^{o}F)
1	fe21f	0.23	0.14	70.79
2	fe21g	0.56	0.59	70.98
3	fe21h	0.62	0.61	71.02
4	ma15g	0.43	0.58	69.53
5	ma15h	0.41	0.49	69.56
6	ma15i	0.38	0.44	69.58
7	ma20f	0.23	0.31	70.50
8	ma20g	0.11	0.19	70.62
9	ma20h	0.23	0.17	70.64

During each of the nine measurements the rail temperature varied over a range ΔT. Plotting the rms residual from the splicing versus the range of rail temperature shows a good correlation. Since the splicing does not account for changes in curvature, the rms residual increases with larger variations in the curvature from position to position.

We averaged the resulting bridge profiles, calculated a mean profile, and calculated the deviation profiles; the difference between the individual profiles and the mean. These differ by quadratics, as expected, since the mean temperature of each measurement was different. We removed the quadratic component from each of the profiles. Three of the quadratic-removed, deviation profiles showed systematic deviations and had larger rms deviations (fe21g,ma15i, and ma20f). We eliminated these three and averaged the remaining six. The rms deviation of the six quadratic-removed, deviation profiles is 1.27 micro-inches. For the final carriage-height error we use the mean of the six carriage-height profiles (fe21f,fe21h,ma15g,ma15h,ma20g,ma20h) and estimate the error on each point to be $1.27 / \sqrt{6} = 0.5$ micro-inches.

Figure 4 shows the carriage-height error. It varies over a range of about 100 micro-inches, and there is a distinct difference between the shape on the negative and positive X portions. There is a short periodicity of about 2.2 inches. On the positive X portion there is a larger wavelength periodicity superimposed. The origins of these are not understood.

If we want have measurements accurate to 1 micro-inch, this error needs to be determined and removed with an accuracy of better than 1%. In addition, the quadratic variations in this error depend on a model of the temperatrue vartiations. These corrections (and the high required accuracy) will be unnecessary when we implement the overhead flat option to the profilometer.

Carriage-Tilt Error

The carriage-tilt profile also affects the measured surface height as follows. A tilt of the carriage displaces the X-position of the probe tip. If the surface being measured is not horizontal, then the X displacement will cause the ball to drop or be pushed up, giving an error in the measured height. The size of the error depends on 1) the carriage tilt, 2) the lever arm between the probe-assembly rotation point and the probe tip, and 3) the slope of the surface being measured.

The carriage has two pads, each of length P and separated by a distance C.

One might think we could calculate the carriage-tilt profile from the carriage-height profile. This cannot be done since some Fourier components of the granite bearing profile have no effect on the height profile, but can have a large effect on the tilt profile. For example, consider a bearing profile that is a sine wave with wavelength equal to C/2. The average of the two pad heights (the carriage height) is constant. The difference of the pad heights (carriage tilt * C) varies by twice the amplitude of the sine wave. We conclude the tilt-profile must be independently measured.

We have used a separate Hewlett-Packard distance-measuring system to measure the carriage tilt. We set a retroreflector at the top of the probe assembly about 14 inches above the retroreflector used to control the carriage X-position. This X-control height is the effective carriage assembly rotation point. The difference in the two measurements divided by the 14 inches, gives the carriage tilt. The resulting carriage-tilt profile is shown in Figure 5. The tilt ranges over about 9 arc seconds.

To calculate the carriage-tilt induced errors in the measured surface we use the following:

surface error = tilt * arm * slope

arm = vertical distance from the X-axis retroreflector to the probe tip.
This depends on the shape and vertical position of the part being measured,
and it varies with X.

slope = the surface slope of the part being measured.
This depends on the shape of the part being measured,
and it varies with X.

The carriage-tilt error profile for the Keck 2 f/15 secondary mirror is shown in Figure 6. The largest effect is at the outer edge of the mirror where the carriage tilt, the lever arm, and the part slope all happen to be high. The maximum error is about 15 micro-inches (=0.38 microns). Note that the error is not symmetric about X = 0, and thus leads to an asymmetry in the measured, uncorrected, profiles.

We also note that implementation of the overhead flat option will have no effect on this carriage-tilt error.

Corrections for Carriage-Height and Carriage-Tilt Errors

The meaurements need to be corrected for the sum of the carriage-height and carriage-tilt effects. A program `tilt_error8`

1) reads a file with the carriage-tilt error (`tilt_error.in1`)
2) reads the carriage-height error (file `beam.shape` produced by FLAT and copied to the file `tilt_error8,in2`),

and 3) writes the sum to its output file (`tilt_error8.out`).

This file is only useful for measurements made at the sample points used for measuring the Keck f/15 secondary. The ouput file must be copied over the `beam.shape` file, since that is the file read by `curvmon`.

tilt_error8.for

```
c  Steve's code processes the measurements of a flat to create a
c  "beam.shape" file  which contains the carriage-height error.
c  This program reads Steve's beam.shape file
c   and adds to it the carriage-tilt error.
c  It then writes an output file "tilt_error.out"  that one can
c   copy over Steve's beam.shape file to have a beam.shape that includes
c   the correction for both effects.
c  This new eam.shape file can then be used by curvmon for fitting the
c   measurements of the Keck 2 secondary.
c This is a temporary fix while we await Steve's availability to
c incorporate the carriage-tilt error in the standard code that creates
c  the beam.shape file.
c
      implicit double precision (a-h,o-z)
      dimension  b(4), c(11), tilt_error(440)
      character*75 aaa(3)
c
        open(unit=2,file='tilt_error8.in1',type='old')
        open(unit=4,file='tilt_error8.in2',type='old')
        open(unit=3,file='tilt_error8.out',type='new')
c
c  READ IN THE CARRIAGE_TILT ERROR (in micro-inches)
      do 30 j =1,440
      read(4,16) tilt_error(j)
   16 format(f12.5)
c     write(3,16) tilt_error(j)
   30 continue
c
c READ IN STEVE'S BEAM.SHAPE FILE (in inches)
      nlines = 3
      do 10 jline = 1,nlines
      read(2,'(75a)') aaa(jline)
      write(3,'(75a)') aaa(jline)
   10 continue
c
      do 20 j=1,440
      read(2,15) (b(m),m=1,4), (c(n),n=1,11)
   15 format(4f15.8,11a1)
c     write(3,15) b,c
c  correct for carriage_tilt error
      b(2) = b(2) + tilt_error(j)/1000000.
      write(3,15) b,c
   20 continue
      stop
      end
```

Appendix 9. Errors in Stitching

We first consider stitching two profiles. The errors in a stitched profile come from
1) statistical errors in the components and
2) systematic errors in the components.

Stitching Two Profiles

Consider two overlapping measured profiles (yA_{meas} and yB_{meas}), where

A: $-\Delta x(M-1)/2 < x < +\Delta x(N-1)/2$

B: $-\Delta x(N-1)/2 < x < +\Delta x(M-1)/2$

with M uniformly-spaced points in the full range,
and N uniformly-spaced points in the overlapping range,
and M,N are odd

The overlap region is $-\Delta x(N-1)/2 < x < +\Delta x(N-1)/2$
symmetric about a point at $x = 0$

To stitch these together we
1. fit in the over lap region
   - yA_{meas} to a line $yA = A1 + A2\, x$
   - yB_{meas} to a line $yB = B1 + B2\, x$
2. subtract each line from the associated data over the full span of each
   - $yA_{meas}' = yA_{meas} - yA$
   - $yB_{meas}' = yB_{meas} - yB$
3. average the resulting spans in the overlap region
   - $yABmeas' = (yA_{meas}' + yB_{meas}')/2$

Statistical Errors on Components

A Least squares fit to a straight line $y(x) = a_1 + a_2 x$ gives

$$a_1^* = \frac{g_1 s_{22} - g_2 s_{12}}{D} \qquad a_2^* = \frac{g_2 s_{11} - g_1 s_{12}}{D}$$

$$(s_{11}, s_{12}, s_{22}) \equiv \frac{\Sigma\,(1, x_i, x_i^2)}{\sigma_i^2} \qquad (g_1, g_2) \equiv \frac{\Sigma\,(1, x_i) y_i}{\sigma_i^2}$$

$$D \equiv s_{11} s_{22} - s_{12}^2$$

The covariance matrix of the fitted parameters $\begin{matrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{matrix} = \frac{1}{D} \begin{matrix} s_{22} & -s_{12} \\ -s_{12} & s_{11} \end{matrix}$

Note the variance does not depend on the y_i ; only on the x values and σ_i

We fit N points in the overlap region of two scans. Consider the fit to one of the scans.

Assume odd N and $x_i = (i - \frac{N+1}{2})\, \Delta x$.

Assume all $\sigma_i = \sigma$ and use $\sum_1^N i = (N/2)(N+1)$ $\qquad \sum_1^N i^2 = (N/6)(2N^2 + 3N + 1)$

$$S_{11} = N \frac{1}{\sigma^2} \qquad S_{12} = \Sigma x_i/\sigma^2 = 0 \qquad S_{22} = \Sigma x_i^2/\sigma^2 = (\frac{\Delta x}{\sigma})^2 \, [\frac{N(N^2-1)}{12}]$$

$$V_{11} = \frac{\sigma^2}{N} \qquad\qquad V_{22} = \frac{1}{S_{22}} = \frac{\sigma^2}{\Delta x^2} [\frac{12}{N(N^2-1)}]$$

$$\Rightarrow \text{ at any x } \quad (\delta y)^2 = \frac{\sigma^2}{N} [\, 1 + \frac{12x^2}{\Delta x^2\, (N^2-1)} \,] \equiv T^2$$

Error on each point on the line $\qquad \delta yA = \delta yB = T$

Assume errors on measured values $\delta yA_{meas} = \delta yB_{meas} = \sigma$

$dyA_{meas}' = [\delta yA_{meas}{}^2 + (\delta Ay)^2]^{1/2} = (\sigma^2 + T^2)^{1/2}$

$dyB_{meas}' = [\delta yB_{meas}{}^2 + (\delta By)^2]^{1/2} = (\sigma^2 + T^2)^{1/2}$

In the overlap region we average the two

$$\delta yAB' = (1/\sqrt{2})\ (\sigma^2 + T^2)^{1/2} = (1/\sqrt{2})\ (\sigma^2 + \frac{\sigma^2}{N} + \frac{12x^2\sigma^2}{\Delta x^2\, N(N^2-1)})^{1/2}$$

We always have $N >> 1$

$$\delta yAB' = (\sigma/\sqrt{2})\ (1 + \frac{12x^2}{\Delta x^2\, N^3})^{1/2}$$

Let $R(x,N) \equiv (1 + \frac{12x^2}{\Delta x^2\, N^3})$

For $-\Delta x(M-1)/2 < x < +\Delta x(N-1)/2 \qquad (\delta y_{out})^2 = \sigma^2\, R(x,N)$

For $-\Delta x(N-1)/2 < x < +\Delta x(N-1)/2 \qquad (\delta y_{in})^2 = \frac{\sigma^2}{2}\, R(x,N)$

For $-\Delta x(N-1)/2 < x < +\Delta x(M-1)/2 \qquad (\delta y_{out})^2 = \sigma^2\, R(x,N)$

Now take the rms error over the full span $-(M-1)/2\, \Delta x$ to $+(M-1)/2\, \Delta x$.

Conclusion: $\quad$ **rms** $\boldsymbol{\delta y = \sigma\, [\, 1 + \frac{M^2}{N^3} - \frac{N}{2M}\,]^{1/2}}$

For the 40-inch flat calibrations $N = 59$ and $M = 94$ $\quad$ rms $\delta y = 0.85\, \sigma$.

This is 20% larger than simply $\sigma / \sqrt{2}$.

For the 20-inch flat calibrations $N = 39$ and $M = 120$ $\quad$ rms $\delta y = 1.04\, \sigma$.

This is 47% larger than simply $\sigma / \sqrt{2}$.

Systematic Errors on Components

Consider a quadratic term in the two profiles. It is easiest to assume continuous functions.

A Least squares fit to a straight line $y(x) = a_1 + a_2 x$ gives

$$a_1^* = \frac{g_1 S_{22} - g_2 s_{12}}{D} \qquad a_2^* = \frac{g_2 s_{11} - g_1 s_{12}}{D}$$

$$(s_{11}, s_{12}, s_{22}) \equiv \frac{1}{\sigma^2} \int (1, x, x^2)\, dx \qquad (g_1, g_2) \equiv \frac{1}{\sigma^2} \int (1, x) y\, dx$$

$$D \equiv s_{11} s_{22} - s_{12}^2$$

For $-a < x < a$

$$s_{11} = 2a \qquad s_{12} = 0 \qquad s_{22} = \frac{2a^3}{3} \qquad D = \frac{4a^4}{3} \qquad \sigma = 1$$

$$a_1 = g_1 / 2a \qquad a_2 = g_2 / \beta$$

Let $yA_{meas} = -\alpha (x + x_0)^2 \qquad yB_{meas} = \alpha (x - x_0)^2 \qquad x_0 \equiv (b + a)/2$

For B: $$a_1 = \frac{\alpha}{6a} [(a - x_0)^3 + (a + x_0)^3] = \frac{\alpha}{3} [a^2 + 3x_0^2] = \alpha\tau$$

where $\tau \equiv \frac{1}{3} [a^2 + 3x_0^2]$

$$a_2 = -2 x_0 \alpha$$

$$yA = -\alpha\tau - 2 \alpha x_0 x \qquad yB = +\alpha\tau - 2 \alpha x_0 x$$

$$yA_{meas}' = -\alpha (x + x_0)^2 + \alpha\tau + 2 \alpha x_0 x = -\alpha [x^2 + x_0^2 - \tau] = -\alpha [x^2 - a^2/3]$$

$$yB_{meas}' = +\alpha (x - x_0)^2 - \alpha\tau + 2 \alpha x_0 x = +\alpha [x^2 + x_0^2 - \tau] = +\alpha [x^2 - a^2/3]$$

In the overlap region

$$yAB_{meas}' = 0$$

Yielding

$$-b < x < -a \qquad yA_{meas}' = -\alpha [x^2 - a^2/3]$$

$$-a < x < a \qquad yAB_{meas}' = 0$$

$$a < x < b \qquad yB_{meas}' = +\alpha [x^2 - a^2/3]$$

Take out overall piston and tilt , then calculate the rms.

For $-b < x < b$

$$S_{11} = 2b \qquad S_{12} = 0 \qquad S_{22} = \frac{2b^3}{3} \qquad D = \frac{4b^4}{3} \qquad \sigma = 1$$

$$G_1 = \int_{-b}^{-a} -\alpha [x^2 - a^2/3]\, dx + \int_{a}^{b} +\alpha [x^2 - a^2/3] dx = 0$$

$$G_2 = \int_{-b}^{-a} -\alpha [x^2 - a^2/3] x\, dx + \int_{a}^{b} +\alpha [x^2 - a^2/3] x\, dx$$

$$= 2\alpha \int_a^b [x^3 - a^2x/3]\, dx = \alpha\, [(b^4 - a^4)/2 - (a^2/3)(b^2 - a^2)\,]$$

$$A_1 = 0$$

$$A_2 = \frac{\alpha}{4b^3}\ [\,3u^4 - 2u^2a^2]\ \Big|_a^b = \frac{\alpha}{4b^3}\,[3(b^4 - a^4) - (2a^2)(b^2 - a^2)]$$

Yielding

$-b < x < -a$	yA_{meas}'	$= -\alpha\,[x^2 - a^2/3]\ - A_2\, x$
$-a < x < a$	yAB_{meas}'	$= -A_2\, x$
$a < x < b$	yB_{meas}'	$= +\alpha\,[x^2 - a^2/3]\ - A_2\, x$

dimensions $[A_2] = 1$ $\quad [\alpha] = m^{-1}$

Now calculate the rms, first calculate the sum of the squared function, S.

$$S = \int_{-b}^{-a} (-\alpha[x^2 - a^2/3] - A_2\, x\,)^2\, dx + \int_a^b (+\alpha\,[x^2 - a^2/3] - A_2\, x\,)^2\, dx + \int_{-a}^{a} (-A_2\, x\,)^2\, dx$$

$$\mathbf{S = 2 \int_a^b (\,\alpha\ [x^2 - a^2/3]\ - A_2\ x\,)^2\, dx + \int_{-a}^{a} (\,A_2\ x\,)^2\, dx}$$

$$S = +2\alpha^2[\,u^5/5 - 2a^2u^3/9\ + a^4u\,/9\,]\ \Big|_a^b\ - \frac{2A_2{}^2b^3}{3}$$

$$\text{rms} = [S\,/\,2b\,]^{1/2}$$

For a single profile over the range $r = (a + b)/2 \qquad z = \alpha r^2\,(x/r)^2$

The rms about the mean $= \dfrac{2\alpha r^2}{3\sqrt{5}} = 0.298\ \alpha r^2$

For the random sum of two completely overlapping profiles,

the rms $= \dfrac{\sqrt{2}\alpha r^2}{3\sqrt{5}} = 0.211\ \alpha r^2$

For the random sum of three completely overlapping profiles,

the rms $= \dfrac{2\alpha r^2}{3\sqrt{15}} = 0.172\ \alpha r^2$

Example:

Consider profiles on the 40-inch flat for the Keck secondary beam shape measurements.

We use three overlapping profiles. They each span 37.43 inches and cover the ranges

A: −2.96 to −40.88 inches

B: −11.94 to −49.37 inches

C: −20.42 to −58.35 inches

For profiles A and B
a = 14.5 inches
b = 23.2 inches
r = (a + b)/2 = 18.7 inches

For profiles A and C
a = 10.2 inches
b = 27.7 inches
r = 19.0 inches

Using these in the equations above gives

$A_2 = \alpha$ 11.984 inches $\qquad$ $A_2 = \alpha$ 18.77 inches

$S = \alpha^2\, 4.394 \times 10^5$ inches^3 $\qquad$ $S = \alpha^2\, 6.15 \times 10^5$ inches^3

rms = $0.287\ \alpha r^2$ $\qquad$ rms = $0.292\ \alpha r^2$

As expected these are smaller than the rms for a single profile (0.298), and larger than two completely-overlapping profiles (0.211).

We estimate that for our three partially-overlapping profiles, the rms (not calculated) will be ~ $0.29\ \alpha r^2$.

Measurements described in Appendix 4 give

$\delta k = -130$ microns / ^{o}F on the Keck secondary

Over the secondary diameter (58 inches) the sag = 1.51 microns / ^{o}F

Over a span of 37.43 inches, sag = 0.63 microns / ^{o}F = 24.8 micro-inches / ^{o}F

This implies αr^2 = 24.8 micro-inches / ^{o}F

Thus $0.29\ \alpha r^2$ = 7.2 micro-inches / ^{o}F

For the 40-inch flat measurements made in Sept 94, there were ~0.3 ^{o}F rms variations from profile to profile within the three profiles.

This implies that for three stitched profiles the

rms = 0.3 x 7.2 = 2.2 micro-inches (= 0.055 microns).

Appendix 10. Reference Surfaces

For a reference profile we use the 20-inch flat, the 40-inch flat, or the 22-inch sphere. We summarize here our knowledge of these surfaces from interferometric measurements.

20-inch flat (Zerodur)

This a rectangular piece (12.5 x 16.5 x 3 inches). The diagonal provides a 20-inch reference profile, hence the name 20-inch flat. It was polished flat and tested by Zygo (203-347-8506, Laurie Erdos (X503) - Optics Administrator, Phil Armitage - Sales Engineer, Lars Selberg, Denny Neely - QC Manager).

The flat was tested standing on its long edge opposite the notch. The results are the average of 7 phase maps using phase-measuring interferometery (18-inch aperture). The three positions show different surfaces, and Neely confirms that the differences are due to measurement noise. The results are summarized as follows.

wavefront

rms error = 0.0087 waves = 0.0055 microns rms
power in wavefront= 0.008 ± 0.007 microns

surface

rms surface error = 2.7 nanometers rms
power = 4 ± 4 nanometers

Zygo tested the piece on edge. When we use the flat for a reference profile we support it on three points. Dave Cowley has used ANSYS to calculate the sag of the surface under its own weight. Figure A10-1 below shows the calculated sag in micro-inches. The maximum sag is 2.3 micro-inches = 0.058 microns.

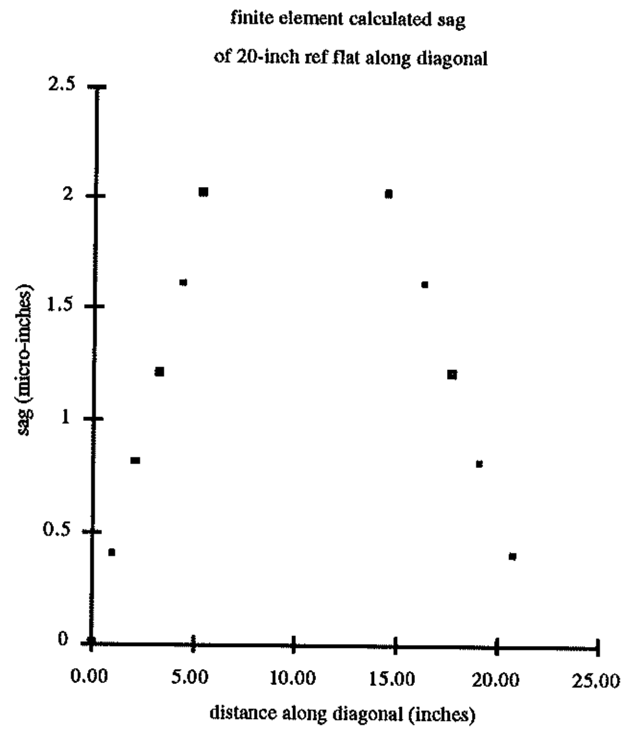


Figure A10-1. Calculated Sag of the 22-inch Flat

22-inch sphere (Pyrex)

The sphere has a diameter = 22 inches and thickness = 5 1/8 inches

An early measurement (23 Oct 1986) of the concave spherical surface gave a radius 4.538 meters and an estimated "figure quality" of $\lambda/20$. The mirror was tested on edge; supported on two felt-padded points with a "V" support and retained by 3 clips.

23 September 1994 we measured a single diameter four times with the profilometer. The fitted radius of curvature is 4.546 meters with a statistical error of 0.0001 meters. The rms difference of each profile from a sphere is 0.033 microns (~ $\lambda/20$). The reference for this analysis was the 40-inch flat. The 0.009 meter difference between the 1986 and 1994 measurements is too large to be explained by uncertainties in the 40-inch flat (they imply about 0.0001 meters in 22-inch sphere) and remains unexplained.

Comparison of the measured profile with the carriage-tilt error shows there is substantial (but not complete) correlation between the two. We do not have a direct measurement of the carriage-tilt error at the time of the 23 September 1994 profilometer measurement. Substracting the current carriage-tilt error by eye from the measured 22-inch sphere profile gives a difference of about 2 micro-inches peak-to-valley (= 0.050 microns) and about 0.6 micro-inches rms (= 0.015 microns) .

On 21 December 1994 we made an interferometric measurement of the sphere using the Zymod (phasing-measuring) upgrade to the Zygo interferometer. The profile along the same diameter measured with profilometer matched a sphere extremely well. The peak-to-valley deviation was 0.030 microns, and the rms deviation was 0.005 microns.

The difference between the profilometer-measured and interferometer-measured profile is dominated by the systematic errors in the profilometer, about 0.015 microns rms.

40-inch flat (Pyrex)

Radius of Curvature

Some tears ago Dave Hilyard made a Ritchey-Common test of the 40-inch flat using the 22-inch sphere.

The measured sagitta of the 40-inch flat is based on an assumed radius of curvature for the 22-inch sphere of 178.66 inches and a measured difference between the tangential and sagittal foci of 0.023 inches. The sag of the 40-inch flat was calculated

$$h = 0.03125 \left(\frac{D}{R}\right)^2 \frac{\cos\phi}{\sin^2\phi} \Delta R \frac{R}{R - S}$$

$D = 22.00$ inches $R = 178.66$ inches $\phi = 63$ deg 25 min
$\cos\phi = 0.45$ $\sin^2\phi = 0.80$ $S = 27.5$ inches
$\Delta R = 77.012 - 76.982$ inches $= 0.030$ inches.

These give h = 9.5 micro-inches = 0.24 microns. The error on this is large because there is an inconsistency in the written record. We estimate ΔR (and h) is known to $\pm 25\%$. Thus the sag over the 22-inch diameter is

h (sag over 22-inch diameter) = 0.24 ± 0.06 microns.

Scaled to a sag over the 40-inch diameter gives

sag over 40-inch diameter = 0.79 ± 0.20 microns.

The description suggests the 40-inch flat is concave.

On 5 Oct 94 we used a spherometer to compare directly the sags of the 20-inch and 40-inch flats. The data and results are summarized in the spreadsheet below.

Spreadsheet - Spherometer Comparison of the 40-inch and 20-inch flats
DH and TM 4Oct94

The distance between the feet is 18.00 ± 0.01 inches.
We tied the cable to the outrigger foot and used gloves to reduce heating.
We tapped the side of the beam after setting it on the glass
to push the probe shaft laterally to the same place each time.

The gauge is a height gauge. A higher surface gives a more positive reading.
The least count on the gauge (an LVDT) is 2 micro-inches.
All measurements are in micro-inches

Measurements were made alternating between the two flats.

	20-inch flat	40-inch flat
	-79	-67
	-78	-67
	-78	-68
	-78	-66
	-78	-68
	-76	-66
	-79	-68
	-79	-70
	-78	-68
	-78	-68
ave=	-78.10	-67.60
stdev=	0.88	1.17
error on mean=	0.28	0.37
difference =	**10.50**	micro-inches
error on difference=	**0.46**	micro-inches

The sign of the difference implies the 40-inch is convex with respect to the 20-inch.

The known sag of the 20-inch is 2.3 micro-inches over 20 inches
(from a finite element analysis of the gravity deflections on its three points).
This corresponds to 1.9 micro-inches over the 18-inch spherometer span.
Thus the 40-inch is convex with a sag of 8.6 ± 0.5 micro-inches over the 18 inches.

Independent confirmation of this was made using a 12-inch flat placed on each surface.
Visually the fringes confirm the 40-inch is convex with respect to the 20-inch.
And visually by about 1/7 to 1/8 wave over the 12 inches.
This implies 7 to 8 micro-inches over 18 inches, agreeing with spherometer's 8.6.

Over 40 inches, the 40-inch flat is convex with sag = 42.5 ± 2.5 micro-inches.

We conclude that the 40-inch flat
1) is convex (This disagrees with the Ritchey-Common writeup.)
and 2) has a sag over 40-inches of 42.5 ± 2.5 micro-inches = 1.08 ± 0.06 microns.

The data analysis of profilometer data where the 40-inch flat was used for the reference has used a concave 40-inch flat with a sag of 24 micro-inches. This was changed to 42.5 micro-inches convex on 6 October 1994 (file loos/inputs/40flat.osd).

Example:
The impact on the Keck secondary was a radius of curvature change

$$\delta k = 0.00030 \text{ meters.}$$

Thermal gradients in the Pyrex mirror will cause a change of curvature. The mirror is supported on an aluminum support, which we expect to rapidly track the ambient temperature. The thermal diffusivity (thermal conductivity / density * heat capacity) for the Pyrex is is about 1 hour. Even with diurnal temperature cycles (~ 10 ^{O}F), we do not expect to develop a gradient in the mirror larger than about 1 ^{O}F. A uniform gradient of 1 ^{O}F induces a sag = 1.3 micro-inches. This is small compared to the observed values.

Profile

The description of the Ritchey-Common test explains the mirror "is believed to be smooth better than $\lambda/10$."

The current data analysis of profilometer data using the 40-inch flat for the reference assumes the flat is perfect other than a finite sag.

The profile was measured on 15 and 16 Sept 94 using the 20-inch flat at five positions along the beam to measure the carriage height profile. The measurements and results are described in a note call "The 40-inch Flat Profile." We summarize the results here. The 40-inch flat profile (with quadratic removed) was measured at three positions along the beam. These profiles agree (the rms deviation from the average is 17 nanometers). The average of the three profiles is shown in Figure A10-2 (in microns). The peak-to_valley is about 0.16 microns (= 6.3 micro-inches), and the rms height of the points is 0.041 microns (=1.6 micro-inches).

40-inch prof quad rmvd ch rev

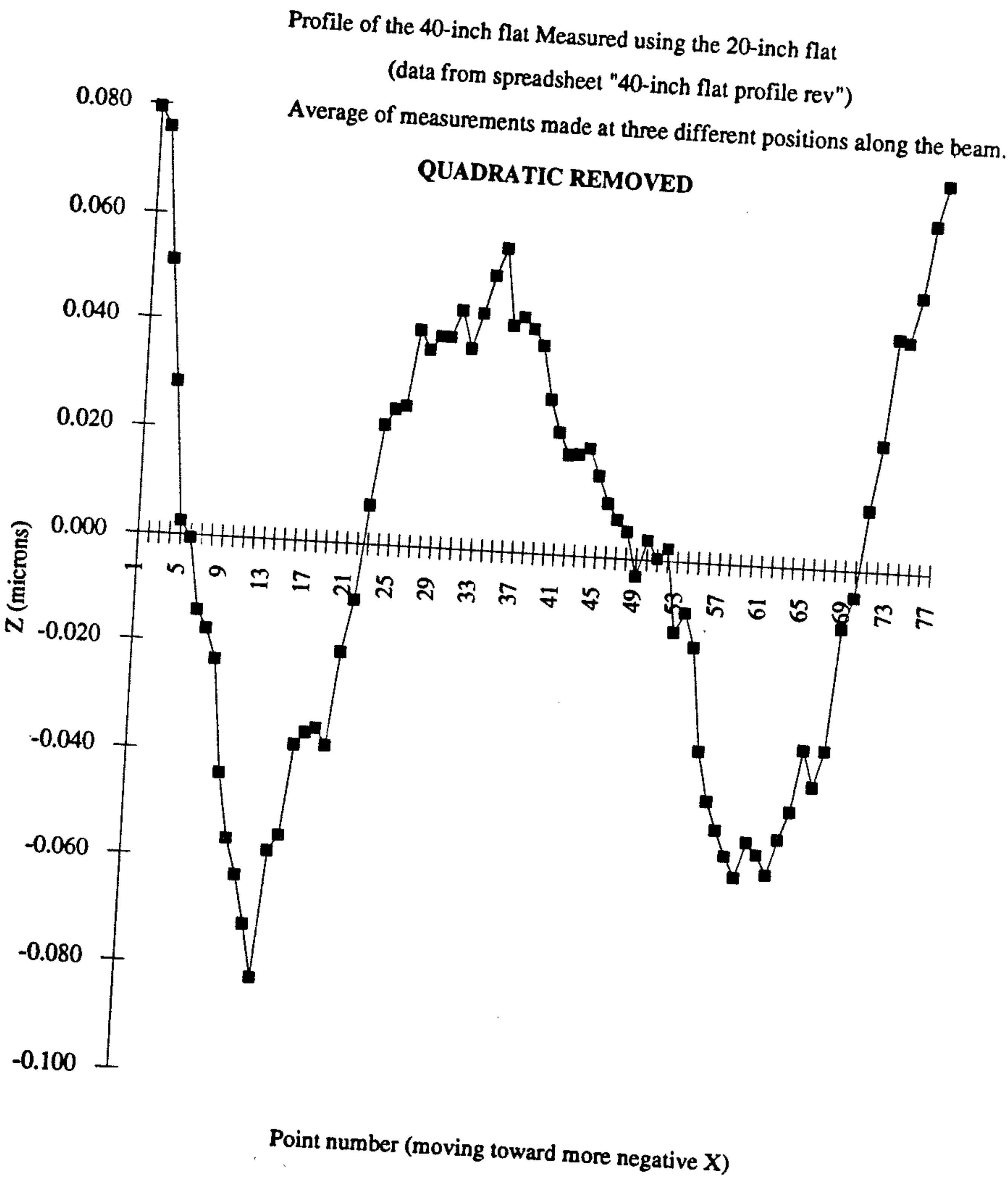


Figure A10-2

Appendix 11. Check of Data Analysis Code

We have checked some aspects of Curvmon by analytically creating input data and comparing the Curvmon output with the known input. The spreadsheet summarizing the details is given below. In particular, we have checked the radius of curvature and conic constant resulting from Curvmon's fit to a single profile. We have also tested the response to a decentered profile. The output agreed with the input for these checks to a fraction of the profilometer least count.

Harland Epps has also provided Curvmon with many other features and outputs. To date these have not been checked with an analytic test like those above. Some of the statistical quantities have been checked independently. However, perhaps the strongest check has been the great many high qualilty surfaces that have been successfully fabricated using this code.

Our standard mode of using the profilometer and the standard manner of running the Curvmon program both assume that the measured surface is azimuthally symmetric. In our standard mode of using the profilometer we measure a set of diameters at different angles and then remove piston and tilt from each profile since the rotation itself will introduce physical tilts of the optic.

In the standard mode of using Curvmon, the cubic term in each profile is removed and the cubic terms from all profiles are then used to calculate a global decenter (t_2 and t_3 of Appendix 7). The residuals from this fit are indicated in the standard output, but the residual cubic is not added back into the profile errors.

As a result of both these, the output of the standard code does not give a true picture of surfaces that vary azimuthally. Harland Epps has run test cases of non-azimuthally symmetric cases. As expected, local surface bumps cause artifacts in the output on the opposite side of the mirror. These are of order 1/3 to 1/2 of the original bump. Thus variations in the profiles should not be believed at the level of the azimuthal variations. However, the output does provide two indications of the presence of azimuthal variations, and **these warnings need to be heeded.**

1) The residual profile plots can show azimuthal variations.
2) The rms residual from the fit to decenters, " RMS(F-C) " will be large if there are azimuthal variations.

Future improvements that would allow accurate measurement of azimuthally-varying surfaces could include:

1) an independent measurement of surface tilt in the rotation from one diameter to the next, using, for example, a central flat on the optic during measurement or a tilt meter attached to the optics during measurement.
2) modification of the Curvmon output to include the residual cubic terms in the residual profiles.

aug 94 checks of curvmon

Units Metric

	Input decen (meters)	k (meters)	K	Output k (meters)	K	DCN (meters)	PST (meters)	TLT (radians)
z1	0.000	4.738	-1.6444414	-4.7380001	-1.6444414	0.000000	0.001000	0.000000
z2	0.000	4.738	-1.6444414	-4.73800084	-1.6444131	0.000000	0.001000	0.000000
v	0.004	4.738	-1.6444414	-4.73800032	-1.6444414	-0.003998	0.000998	0.000844

Units English

	Input decen (inches)	k (inches)	K	CV*1000 (inches^-1)	K	DCN (inches)	PST (inches)	TLT (degrees)
z	0.000000	186.535433	-1.6444414	-5.36091166	-1.6444414	0.0000	0.039370	0.000000
z	0.000000	186.535433	-1.6444414	-5.36091083	-1.6444131	0.0000	0.039370	0.000000
w	0.157480	186.535433	-1.6444414	-5.36091141	-1.6444414	-0.1574	0.039304	0.048351

Decenter Output/Input = -0.99949

TLT = A * decenter, where A = 0.2110

Conclusions

z1 K was held fixed, k was varied — dk corresponds to a sag error of 0.11 counts

z2 k and k varied — dk corresponds to a sag error of 0.90 counts; dK corresponds to a sag error of 0.35 counts

v — output decenter is proportional to input; output agrees with input to ~ 0.0005

PST agrees with (decen)^2/(2k)

TLT agrees with (decen)/(2k)

dsag = (a^2/2k) dk = (1/86) * dk

dsag = 3 (K+1) a^4 /(8k^4) dK = dK/7632

a31 ~ a^3KR/(2k^3)

dz/dR = a^3/(2k^3) = 1/563